
FREE MATH TUTOR • MATHEMATICS TEXTBOOK

10^N

10^N

POWERS OF TEN FROM TINY TO ENORMOUS

Place value • Number names • Scientific notation
Geometry • Cosmic estimates • Finite giants • Infinity

10⁷⁹ = ten quinvigintillion

1 followed by 79 zeros • 80 digits altogether

ILLUSTRATED LESSONS • REFERENCE TABLES • 48 EXERCISES

WORKED ANSWERS • GLOSSARY • SOURCES • INDEX

A guide to this textbook

A power of ten is a compact description of size. This book begins with decimal place value, develops the rules that make calculations dependable, and then follows those rules beyond numbers that can be written out in full.

Section	Pages	What you will learn
Foundations	3-8	Number sets; positive, zero, negative, and fractional exponents; tiny numbers.
Calculation toolkit	9-12	Exponent laws, scientific notation, logarithms, comparison, and digit counts.
Naming and reference atlas	13-20	Short and long scales; every integer power from 0 to 100; centillion.
Powers in the world	21-24	SI prefixes, geometric scaling, time and counting, and a universe estimate.
Beyond familiar numbers	25-28	Googolplexes, towers, Graham's number, TREE(3), and infinity.
Review and practice	29-36	Error checks, 48 exercises, and worked answers.
Reference tools	37-39	Glossary, source notes, and a subject index.

How to use the pages

Read pages 3-13 first. Use the atlas as a reference rather than a list to memorize. Work the exercises on pages 30-33 before consulting the answers. The advanced pages are an introduction; they do not require computing an enormous decimal expansion.

CONVENTIONS

Number names use the United States short scale unless explicitly labeled otherwise. Commas separate groups of three digits. The symbol $\approx$ means approximately equal; $=$ states exact equality. The notation 10^N is a plain-text form of 10^N .

Classroom edition. Teachers and learners may print and share this complete edition for educational use. Explanations, diagrams, examples, and exercises were prepared for this textbook. Source notes distinguish mathematical definitions from measured or model-dependent estimates.

Is there a smallest or greatest number?

There is no greatest finite number. If someone proposes a number M , then $M + 1$ is greater. If M is a positive power of ten, multiplying it by 10 gives the next larger power. A name, a long string of digits, or a famous mathematical construction cannot stop this process.

Set of numbers	Smallest member?	Greatest member?
Positive integers: 1, 2, 3, ...	1	None
Whole numbers: 0, 1, 2, ...	0	None
Integers: ..., -2, -1, 0, 1, ...	None	None
Positive real numbers	None	None
All real numbers	None	None

NO FIRST INTEGER • NO LAST INTEGER



Every proposed endpoint has another number beyond it.

Why positive real numbers have no least member

Take any positive real number x . The number $x/10$ is still positive and is smaller than x . Thus 0.1 is not the smallest positive number: 0.01 is smaller. The same reasoning defeats 10^{-100} , $10^{-1000000}$, and every other proposed positive minimum.

$$0 < x/10 < x \text{ for every } x > 0$$

ZERO IS A BOUNDARY, NOT A POSITIVE NUMBER

Zero is the greatest lower bound of the positive real numbers, but is not a member of that set. Negative numbers continue without a least member: if x is negative, $x - 1$ is even smaller.

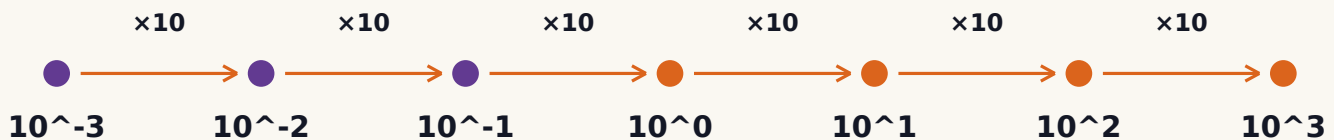
A complete list of every number is impossible. The aim here is to understand representative values, naming systems, and procedures that continue indefinitely.

What 10^N means

In 10^N, the base is 10 and N is the exponent. When N is a positive integer, the expression means the product of N copies of 10. It does not mean 10 × N. For example, 10⁴ = 10 × 10 × 10 × 10 = 10,000, while 10 × 4 = 40.

Power	Repeated multiplication	Value	Digits
10 ¹	10	10	2
10 ²	10 × 10	100	3
10 ³	10 × 10 × 10	1,000	4
10 ⁴	Four factors of 10	10,000	5
10 ⁵	Five factors of 10	100,000	6

EQUAL STEPS IN EXPONENT = EQUAL MULTIPLIERS



Read to the left: divide by 10 at every step.

EXAMPLE 01 | Count the zeros and digits

For 10⁷, write one 1 and seven zeros: 10,000,000.

Zero count = 7. Total digit count = 1 + 7 = 8.

Commas organize the digits; they are not digits themselves.

The rule has a domain

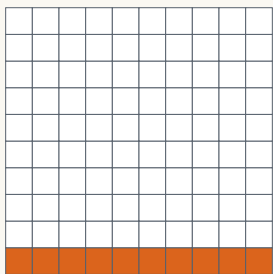
For a nonnegative integer N, 10^N is written as a 1 followed by N zeros, and has N + 1 digits. This description does not apply literally to negative or noninteger exponents. Their meanings follow from reciprocals and roots.

Place value extends in both directions: thousands, hundreds, tens, ones, tenths, hundredths, thousandths. Each adjacent position has ten times or one-tenth the value of its neighbor.

Zero and negative exponents

Divide successive powers by 10: 1,000, 100, 10, 1, 0.1, 0.01. Their exponents fall by one at each step: 3, 2, 1, 0, -1, -2. The pattern explains both the zero exponent and the reciprocal meaning of a negative exponent.

$$10^0 = 1 \text{ and } 10^{-n} = 1/10^n$$



ONE WHOLE = 100 EQUAL SQUARES

One square = $1/100 = 10^{-2}$

One row = $10/100 = 1/10 = 10^{-1}$

Ten rows = $100/100 = 1 = 10^0$

EXAMPLE 02 | Why the zero power is one

$10^3/10^3 = 1$ because any nonzero number divided by itself is 1. Using exponent subtraction gives $10^{3-3} = 10^0$. Therefore $10^0 = 1$, not 0.

EXAMPLE 03 | Decode a negative exponent

$10^{-5} = 1/10^5 = 1/100,000 = 0.00001$.

The 1 is in the fifth decimal place. Four zeros lie between the decimal point and the 1. The zero before the decimal point is written for clarity.

Expression	Value	Sign
10^{-3}	0.001	Positive
-10^3	-1,000	Negative
$(-10)^3$	-1,000	Negative
$(-10)^2$	100	Positive

Do not confuse the sign of an exponent with the sign of the number. Every real power of the positive base 10 is positive. A leading minus sign outside the power negates the whole value.

Fractional and real exponents

Exponents need not be integers. A fractional exponent combines a root with a power. For integers p and q with $q > 0$, the positive base 10 makes the real q th root unambiguous.

$$10^{p/q} = (\text{qth root of } 10)^p$$

EXAMPLE 04 | An exponent halfway between 0 and 1

Let $x = 10^{1/2}$. Squaring gives $x^2 = 10$, so $x = \sqrt{10}$.

Because $3^2 = 9$ and $4^2 = 16$, we know $3 < \sqrt{10} < 4$.

More closely, $3.16^2 = 9.9856$ and $3.17^2 = 10.0489$, so $3.16 < \sqrt{10} < 3.17$. The exact answer remains $\sqrt{10}$.

EXAMPLE 05 | Separate a whole part from a fraction

$10^{3/2} = 10^1 + 1/2 = 10 \times \sqrt{10}$.

The previous bounds imply $31.6 < 10^{3/2} < 31.7$.

$10^{-1/2} = 1/\sqrt{10} = \sqrt{10}/10$.

Exponent	Exact value	Location
0	1	At one
1/2	$\sqrt{10}$	Between 3 and 4
1	10	At ten
3/2	$10\sqrt{10}$	Between 31 and 32
2	100	At one hundred

Beyond fractions

For a real x , one definition is $10^x = e^{x \ln(10)}$. This extends the same continuous, increasing function to irrational exponents such as $\sqrt{2}$ and π . The exponent laws on page 9 remain valid for real exponents with this positive base.

WHERE THE “ZEROS” RULE STOPS

$10^{1/2}$ is not a 1 followed by half a zero. Its exact form is $\sqrt{10}$.

Complex exponents can also be defined, but complex numbers do not have the ordinary real-number ordering used in this textbook.

An atlas of tiny powers

The reciprocal names below follow the short scale. Each negative exponent moves the final 1 one place farther to the right. In the decimal form of 10^{-n} , exactly $n - 1$ zeros stand between the decimal point and that 1.

Power	Read as	Power	Read as
10^{-1}	one tenth	10^{-16}	one ten-quadrillionth
10^{-2}	one hundredth	10^{-17}	one hundred-quadrillionth
10^{-3}	one thousandth	10^{-18}	one quintillionth
10^{-4}	one ten-thousandth	10^{-19}	one ten-quintillionth
10^{-5}	one hundred-thousandth	10^{-20}	one hundred-quintillionth
10^{-6}	one millionth	10^{-21}	one sextillionth
10^{-7}	one ten-millionth	10^{-22}	one ten-sextillionth
10^{-8}	one hundred-millionth	10^{-23}	one hundred-sextillionth
10^{-9}	one billionth	10^{-24}	one septillionth
10^{-10}	one ten-billionth	10^{-25}	one ten-septillionth
10^{-11}	one hundred-billionth	10^{-26}	one hundred-septillionth
10^{-12}	one trillionth	10^{-27}	one octillionth
10^{-13}	one ten-trillionth	10^{-28}	one ten-octillionth
10^{-14}	one hundred-trillionth	10^{-29}	one hundred-octillionth
10^{-15}	one quadrillionth	10^{-30}	one nonillionth

EXAMPLE 06 | Read a very small decimal

0.000000000001 has its 1 in decimal place 12.

Its value is 10^{-12} , or one trillionth.

There are eleven zeros after the decimal point before the 1.

SMALLER THAN THE TABLE

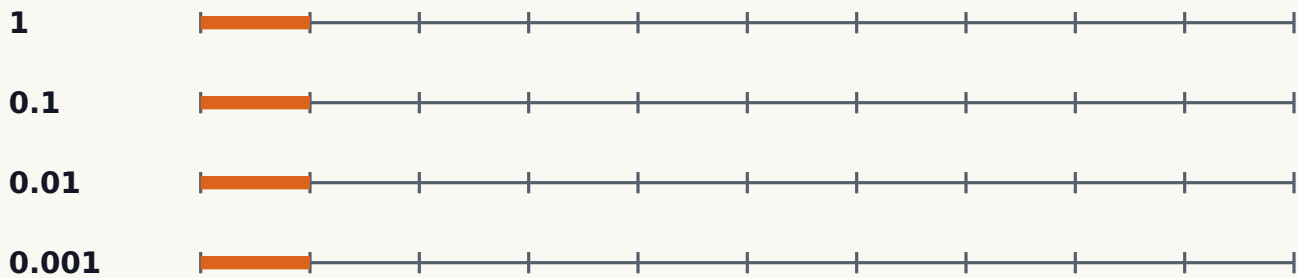
$10^{-100} = 1/\text{googol}$. The positive number $10^{-(10^{100})}$ is $1/\text{googolplex}$.

Dividing either number by 10 produces an even smaller positive number.

Neither is zero.

Hyphens clarify compound denominator names. For any difficult name, “one divided by ten to the n th power” is exact and avoids spelling ambiguity.

Tiny is not the same as zero



Each row magnifies the orange tenth of the row above.

The sequence 1, 1/10, 1/100, 1/1,000, ... approaches zero. Every term is positive. No finite step turns 10^{-n} into zero. A limit describes the behavior of the whole sequence as the index increases without bound.

$$10^{-n} \rightarrow 0 \text{ as } n \rightarrow \infty$$

A mathematical value and a physical scale

The Planck length is about 1.616×10^{-35} metre [3]. It is a physical scale constructed from constants. That value does not establish a smallest positive real number. For example, 10^{-100} metre is a meaningful mathematical length expression even though this book makes no claim that such a length can be measured.

Concept	What it tells you
Small positive real	A value greater than zero; dividing by 10 makes it smaller.
Measurement resolution	The distinctions an instrument or method can reliably make.
Rounding to zero	A display or precision decision, not exact mathematical equality.
Infinitesimal	In certain extended number systems, a positive value smaller than every positive real $1/n$.

STANDARD REAL NUMBERS

In the real numbers, there is no positive infinitesimal: for every $x > 0$, some positive integer n satisfies $1/n < x$. Nonstandard analysis uses a different, extended number system. Do not silently mix its rules with ordinary real arithmetic.

A calculator can show 0 because it has rounded or underflowed. Always check whether the mathematical expression is actually zero before canceling or dividing by it.

The exponent-law toolkit

With the positive base 10, the following rules hold for real exponents. For a general base, extra restrictions may be needed. The examples here are exact arithmetic, independent of how many decimal digits fit on a page. See [1] for a foundational treatment.

Rule	Formula	Example
Product	$10^a \times 10^b = 10^{a+b}$	$10^3 \times 10^4 = 10^7$
Quotient	$10^a / 10^b = 10^{a-b}$	$10^3 / 10^4 = 10^{-1}$
Power of a power	$(10^a)^b = 10^{ab}$	$(10^3)^4 = 10^{12}$
Zero exponent	$10^0 = 1$	$10^5 / 10^5 = 1$
Reciprocal	$10^{-a} = 1/10^a$	$10^{-6} = 1/10^6$
Root	qth root of $10^a = 10^{a/q}$	$\sqrt[4]{(10^8)} = 10^2$

EXAMPLE 07 | Combine operations in order

Simplify $(10^8 \times 10^{-3})/10^2$.

1. Multiply the numerator: $10^8 + (-3) = 10^5$.
2. Divide: $10^5 - 2 = 10^3$.
3. Write the value: 1,000.

EXAMPLE 08 | A negative power of a power

$(10^{-4})^{-2} = 10^{(-4)(-2)} = 10^8$.

The two exponent signs multiply to positive. Equivalently, take the reciprocal of $(10^{-4})^2 = 10^{-8}$.

ADDITION USES A DIFFERENT RULE

$10^3 + 10^3 = 2 \times 10^3 = 2,000$. It is not 10^6 . Exponents add when powers of the same base are multiplied. When terms are added, first factor or align their powers.

A useful habit: name the operation before choosing the rule. Multiply → add exponents. Divide → subtract exponents. Raise a power to a power → multiply exponents.

Scientific notation, step by step

Scientific notation expresses a nonzero real number as $a \times 10^n$, where n is an integer and $1 \leq |a| < 10$. The coefficient a carries the leading digits and the sign; the power of ten carries the scale. Zero is written simply as 0.

EXAMPLE 09 | A large number

Write 58,200,000 in scientific notation.

1. Put the decimal after the first nonzero digit: 5.82.
2. Count seven place-value shifts from 58,200,000 to 5.82.
3. Restore the scale: 5.82×10^7 .

Check: multiplying 5.82 by 10,000,000 gives 58,200,000.

EXAMPLE 10 | A small number

Write 0.0000426 in scientific notation.

1. Use the coefficient 4.26.
2. The first nonzero digit is in the fifth decimal place.
3. Multiply by 10^{-5} : 4.26×10^{-5} .

Check: $4.26/100,000 = 0.0000426$.

Ordinary form	Scientific notation	Reason
-7,500	-7.5×10^3	Keep the sign with the coefficient.
0.008	8×10^{-3}	Three decimal places to the 8.
100,000,000	1×10^8	An exact power of ten.
42×10^5	4.2×10^6	Normalize a coefficient larger than 10.
0.42×10^5	4.2×10^4	Normalize a coefficient smaller than 1.

What a calculator display means

A display such as 5.82E7 usually means 5.82×10^7 . The E here is exponent notation, not Euler's number e . Scientific notation can describe exact integers as well as rounded measurements; the context tells you which is intended.

The values 4.2×10^6 and 4.20×10^6 are mathematically equal. In measurement reporting, the extra trailing zero can communicate one additional significant digit.

Calculate without long strings of zeros

EXAMPLE 11 | Multiply

$$(3 \times 10^7)(4 \times 10^5)$$

1. Multiply coefficients: $3 \times 4 = 12$.
2. Add exponents: $7 + 5 = 12$.
3. Combine: 12×10^{12} .
4. Normalize: $1.2 \times 10^{13} = \text{twelve trillion}$.

EXAMPLE 12 | Divide

$$(8.4 \times 10^{-3}) / (2.1 \times 10^5)$$

1. Divide coefficients: $8.4 / 2.1 = 4$.
2. Subtract exponents: $-3 - 5 = -8$.
3. Combine: $4 \times 10^{-8} = 0.00000004$.

EXAMPLE 13 | Add by aligning powers

$$2.3 \times 10^6 + 4.5 \times 10^5$$

1. Rewrite the second term: $4.5 \times 10^5 = 0.45 \times 10^6$.
2. Add coefficients: $(2.3 + 0.45) \times 10^6$.
3. Result: $2.75 \times 10^6 = 2,750,000$.

EXAMPLE 14 | Subtract

$$7 \times 10^4 - 8 \times 10^3$$

1. Rewrite 8×10^3 as 0.8×10^4 .
2. Subtract coefficients: $(7 - 0.8) \times 10^4$.
3. Result: $6.2 \times 10^4 = 62,000$.

A QUICK SIZE CHECK

Example 11 multiplies two large positive numbers, so its result should exceed each factor. Example 12 divides a small positive number by a large positive number, so the result should be much smaller. Such checks catch many exponent-sign errors.

For measured inputs, carry a few guard digits during the arithmetic and round at the end. Multiplication and division normally depend on significant digits; addition and subtraction depend on the precision of aligned decimal places. Exact classroom numbers need no artificial rounding.

Compare sizes and count digits

For positive normalized scientific notation, compare the exponents first. If they match, compare the coefficients. A larger exponent wins even when its coefficient looks smaller: $1.1 \times 10^9 > 9.9 \times 10^8$.

EXAMPLE 15 | How many times larger?

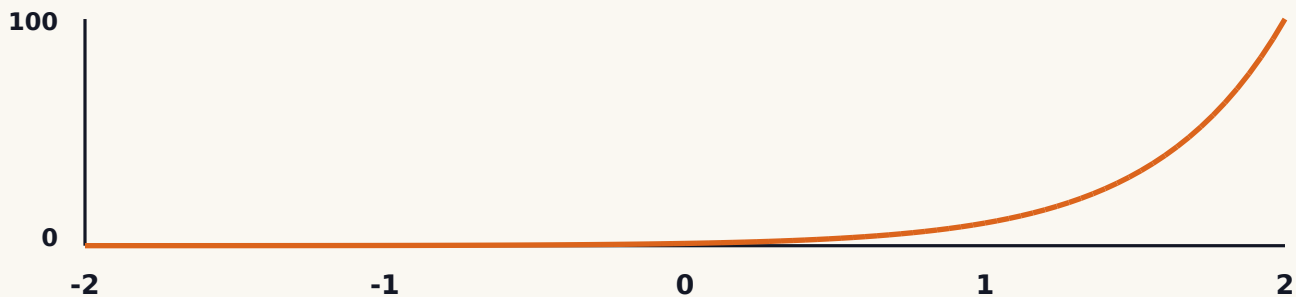
$$10^{79}/10^{75} = 10^{79-75} = 10^4 = 10,000.$$

Thus 10^{79} is ten thousand times 10^{75} . The difference of four in the exponents is a multiplicative gap, not a difference of four in the values.

$$\log_{10}(10^x) = x$$

A base-ten logarithm answers: “What exponent of 10 gives this positive number?” Thus $\log_{10}(1,000) = 3$ and $\log_{10}(0.001) = -3$. A standard logarithmic axis represents positive values; zero and negative values have no real base-ten logarithm.

$y = 10^x$ rises rapidly; it never equals 0.



EXAMPLE 16 | Count digits without writing the number

For a positive integer K, $\text{digits}(K) = \text{floor}(\log_{10} K) + 1$.

Since $10^{12} \leq 7 \times 10^{12} < 10^{13}$, the integer 7×10^{12} has 13 digits. No calculator is needed: locate it between consecutive powers of ten.

TWO USES OF “ORDER OF MAGNITUDE”

In this book, an exponent gap of one means a factor of 10. Some references assign an order of magnitude using the nearest power of ten, while others use the exponent in scientific notation. State the convention when a boundary case matters.

For negative numbers, reverse the size intuition: -10^9 is less than -10^6 . Its absolute value is larger, but its position on the real number line is farther left.

How large-number names are built

English number names are conventions layered on place value. Beyond a thousand, the United States short scale assigns a new -illion name every three powers of ten. Different naming scales developed historically, so a bare word such as billion can be ambiguous across older texts and languages [2].

Name	Short scale used here	Long scale
million	10^6	10^6
billion	10^9	10^{12}
trillion	10^{12}	10^{18}
quadrillion	10^{15}	10^{24}
centillion	10^{303}	10^{600}

Index the -illion names

Short scale: the k th -illion = 10^{3k+3}

Million has $k = 1$, billion $k = 2$, and trillion $k = 3$. Between each named base are its tenfold and hundredfold multiples. At the next thousandfold increase, advance to the next name. The long-scale k th -illion instead corresponds to 10^{6k} .

EXAMPLE 17 | Find the name for 10^{79}

Write $79 = 3(25) + 3 + 1$.

The base exponent 78 belongs to $k = 25$: quinvigintillion.

The remaining 1 in the exponent contributes a factor of 10.

Therefore $10^{79} = \text{ten quinvigintillion}$.

A RELIABLE NAMING ALGORITHM

For $n \geq 6$, let $k = \text{floor}(n/3) - 1$ and $r = n - 3k - 3$.

Then r is 0, 1, or 2. Read 10^n as one, ten, or one hundred of the k th -illion, respectively.

Beyond familiar names, spellings vary among naming extensions. This book uses a consistent common extended spelling, including quinvigintillion. Powers of ten remain exact regardless of spelling. “Zillion” and “gazillion” have no fixed numerical values.

Power atlas: 10⁰ through 10²⁰

From one to one hundred quintillion. The zero count is n and the digit count is $n + 1$ for every row. These are exact positive integers; commas and typography do not change their values.

Power	Name in the United States short scale	Zeros	Digits
10 ⁰	One	0	1
10 ¹	Ten	1	2
10 ²	One hundred	2	3
10 ³	One thousand	3	4
10 ⁴	Ten thousand	4	5
10 ⁵	One hundred thousand	5	6
10 ⁶	One million	6	7
10 ⁷	Ten million	7	8
10 ⁸	One hundred million	8	9
10 ⁹	One billion	9	10
10 ¹⁰	Ten billion	10	11
10 ¹¹	One hundred billion	11	12
10 ¹²	One trillion	12	13
10 ¹³	Ten trillion	13	14
10 ¹⁴	One hundred trillion	14	15
10 ¹⁵	One quadrillion	15	16
10 ¹⁶	Ten quadrillion	16	17
10 ¹⁷	One hundred quadrillion	17	18
10 ¹⁸	One quintillion	18	19
10 ¹⁹	Ten quintillion	19	20
10 ²⁰	One hundred quintillion	20	21

READ THE THREE-STEP RHYTHM

One → ten → one hundred of the same named unit. The next row begins a new unit one thousand times the preceding base. For example, one million → ten million → one hundred million → one billion.

Reference convention: short-scale powers and common extended -illion names [2]. Every row is generated from the base-ten place-value rule; no row denotes infinity.

Power atlas: 10²¹ through 10⁴¹

From sextillion to one hundred duodecillion. The zero count is n and the digit count is $n + 1$ for every row. These are exact positive integers; commas and typography do not change their values.

Power	Name in the United States short scale	Zeros	Digits
10 ²¹	One sextillion	21	22
10 ²²	Ten sextillion	22	23
10 ²³	One hundred sextillion	23	24
10 ²⁴	One septillion	24	25
10 ²⁵	Ten septillion	25	26
10 ²⁶	One hundred septillion	26	27
10 ²⁷	One octillion	27	28
10 ²⁸	Ten octillion	28	29
10 ²⁹	One hundred octillion	29	30
10 ³⁰	One nonillion	30	31
10 ³¹	Ten nonillion	31	32
10 ³²	One hundred nonillion	32	33
10 ³³	One decillion	33	34
10 ³⁴	Ten decillion	34	35
10 ³⁵	One hundred decillion	35	36
10 ³⁶	One undecillion	36	37
10 ³⁷	Ten undecillion	37	38
10 ³⁸	One hundred undecillion	38	39
10 ³⁹	One duodecillion	39	40
10 ⁴⁰	Ten duodecillion	40	41
10 ⁴¹	One hundred duodecillion	41	42

READ THE THREE-STEP RHYTHM

One → ten → one hundred of the same named unit. The next row begins a new unit one thousand times the preceding base. For example, one million → ten million → one hundred million → one billion.

Reference convention: short-scale powers and common extended -illion names [2]. Every row is generated from the base-ten place-value rule; no row denotes infinity.

Power atlas: 10⁴² through 10⁶²

From tredecillion to one hundred novemdecillion. The zero count is n and the digit count is $n + 1$ for every row. These are exact positive integers; commas and typography do not change their values.

Power	Name in the United States short scale	Zeros	Digits
10 ⁴²	One tredecillion	42	43
10 ⁴³	Ten tredecillion	43	44
10 ⁴⁴	One hundred tredecillion	44	45
10 ⁴⁵	One quattuordecillion	45	46
10 ⁴⁶	Ten quattuordecillion	46	47
10 ⁴⁷	One hundred quattuordecillion	47	48
10 ⁴⁸	One quindecillion	48	49
10 ⁴⁹	Ten quindecillion	49	50
10 ⁵⁰	One hundred quindecillion	50	51
10 ⁵¹	One sexdecillion	51	52
10 ⁵²	Ten sexdecillion	52	53
10 ⁵³	One hundred sexdecillion	53	54
10 ⁵⁴	One septendecillion	54	55
10 ⁵⁵	Ten septendecillion	55	56
10 ⁵⁶	One hundred septendecillion	56	57
10 ⁵⁷	One octodecillion	57	58
10 ⁵⁸	Ten octodecillion	58	59
10 ⁵⁹	One hundred octodecillion	59	60
10 ⁶⁰	One novemdecillion	60	61
10 ⁶¹	Ten novemdecillion	61	62
10 ⁶²	One hundred novemdecillion	62	63

READ THE THREE-STEP RHYTHM

One → ten → one hundred of the same named unit. The next row begins a new unit one thousand times the preceding base. For example, one million → ten million → one hundred million → one billion.

Reference convention: short-scale powers and common extended -illion names [2]. Every row is generated from the base-ten place-value rule; no row denotes infinity.

Power atlas: 10⁶³ through 10⁸³

From vigintillion to one hundred sexvigintillion. The zero count is n and the digit count is $n + 1$ for every row. These are exact positive integers; commas and typography do not change their values.

Power	Name in the United States short scale	Zeros	Digits
10 ⁶³	One vigintillion	63	64
10 ⁶⁴	Ten vigintillion	64	65
10 ⁶⁵	One hundred vigintillion	65	66
10 ⁶⁶	One unvigintillion	66	67
10 ⁶⁷	Ten unvigintillion	67	68
10 ⁶⁸	One hundred unvigintillion	68	69
10 ⁶⁹	One duovigintillion	69	70
10 ⁷⁰	Ten duovigintillion	70	71
10 ⁷¹	One hundred duovigintillion	71	72
10 ⁷²	One trevigintillion	72	73
10 ⁷³	Ten trevigintillion	73	74
10 ⁷⁴	One hundred trevigintillion	74	75
10 ⁷⁵	One quattuorvigintillion	75	76
10 ⁷⁶	Ten quattuorvigintillion	76	77
10 ⁷⁷	One hundred quattuorvigintillion	77	78
10 ⁷⁸	One quinvigintillion	78	79
10 ⁷⁹	Ten quinvigintillion	79	80
10 ⁸⁰	One hundred quinvigintillion	80	81
10 ⁸¹	One sexvigintillion	81	82
10 ⁸²	Ten sexvigintillion	82	83
10 ⁸³	One hundred sexvigintillion	83	84

READ THE THREE-STEP RHYTHM

One → ten → one hundred of the same named unit. The next row begins a new unit one thousand times the preceding base. For example, one million → ten million → one hundred million → one billion.

Reference convention: short-scale powers and common extended -illion names [2]. Every row is generated from the base-ten place-value rule; no row denotes infinity.

Power atlas: 10⁸⁴ through 10¹⁰⁰

From septenvigintillion to a googol. The zero count is n and the digit count is $n + 1$ for every row. These are exact positive integers; commas and typography do not change their values.

Power	Name in the United States short scale	Zeros	Digits
10 ⁸⁴	One septenvigintillion	84	85
10 ⁸⁵	Ten septenvigintillion	85	86
10 ⁸⁶	One hundred septenvigintillion	86	87
10 ⁸⁷	One octovigintillion	87	88
10 ⁸⁸	Ten octovigintillion	88	89
10 ⁸⁹	One hundred octovigintillion	89	90
10 ⁹⁰	One novemvigintillion	90	91
10 ⁹¹	Ten novemvigintillion	91	92
10 ⁹²	One hundred novemvigintillion	92	93
10 ⁹³	One trigintillion	93	94
10 ⁹⁴	Ten trigintillion	94	95
10 ⁹⁵	One hundred trigintillion	95	96
10 ⁹⁶	One untrigintillion	96	97
10 ⁹⁷	Ten untrigintillion	97	98
10 ⁹⁸	One hundred untrigintillion	98	99
10 ⁹⁹	One duotrigintillion	99	100
10 ¹⁰⁰	Ten duotrigintillion (a googol)	100	101

TWO NAMES, ONE NUMBER

10¹⁰⁰ is one googol. In the extended short scale it is also ten duotrigintillion, because one duotrigintillion is 10⁹⁹. One tretrigintillion is 10¹⁰², so a googol lies between those two named bases.

Use exponent subtraction for every comparison in the atlas. For instance, $10^{100}/10^{99} = 10$, whereas $10^{102}/10^{100} = 100$. The length of a spoken name is not a measure of the number's size.

Spotlight: ten quinvigintillion

$10^{79} = 10 \times 10^{78}$

The name ten quinvigintillion is correct in the United States short scale. The number is exactly a 1 followed by 79 zeros. There are 80 digits altogether. Written below, the number continues from line to line; the line breaks do not add or remove digits.

THE FULL DECIMAL EXPANSION

10,000,000,000,000,000,000,000,000,
000,000,000,000,000,000,000,000,000,
000,000,000,000,000,000,000,000,000

Power	Name	Relative to 10^79
10^75	One quattuorvigintillion	1/10,000
10^78	One quinvigintillion	1/10
10^79	Ten quinvigintillion	1
10^80	One hundred quinvigintillion	10
10^81	One sexvigintillion	100

EXAMPLE 18 | Check the written expansion

79 = 3 × 26 + 1. Therefore the comma-grouped expansion has an initial “10” followed by 26 groups of “000.”

Digits: 2 + 26 × 3 = 80.

Zeros: 1 + 26 × 3 = 79.

This check is safer than counting an uninterrupted string by eye.

What about the observable universe?

The popular comparison to the number of atoms in the observable universe is a scale estimate. An ordinary-matter nucleon count of roughly 10^80 is more commonly used. An atom can contain many nucleons, and much cosmic matter is ionized. Page 24 shows how a simplified estimate reaches this broad scale.

An exact integer and an uncertain physical estimate are different kinds of statement. 10^79 is exactly ten quinvigintillion even if a cosmological estimate changes.

Beyond a googol: toward centillion

The -illion pattern continues far beyond the atlas. The table gives selected milestones, not a final endpoint. A named number is useful only when its naming convention or mathematical definition is clear.

Index k	Short-scale name	Power	Digits
30	One trigintillion	10^{93}	94
31	One untrigintillion	10^{96}	97
32	One duotrigintillion	10^{99}	100
33	One tretrigintillion	10^{102}	103
40	One quadragintillion	10^{123}	124
50	One quinquagintillion	10^{153}	154
60	One sexagintillion	10^{183}	184
70	One septuagintillion	10^{213}	214
80	One octogintillion	10^{243}	244
90	One nonagintillion	10^{273}	274
100	One centillion	10^{303}	304

EXAMPLE 19 | Why centillion is 10^{303}
Centi- corresponds to index $k = 100$ in this naming pattern.
Exponent = $3k + 3 = 3(100) + 3 = 303$.
Thus one short-scale centillion = 10^{303} : a 1 followed by 303 zeros, with 304 digits.

CENTILLION IS LARGER THAN A GOOGOL
 $10^{303}/10^{100} = 10^{203}$. A centillion is 10^{203} times a googol.
But a googolplex = $10^{(10^{100})}$, so its exponent is itself a googol. That is enormously larger than the exponent 303.

Extended name systems can build labels beyond centillion, and different authors use different spelling rules. Scientific notation does not depend on these choices. “Ten to the ten-thousandth power” unambiguously specifies 10^{10000} , whether or not one knows an -illion label for it.

Do not infer the exponent by the length of the word. The indexed formula is the reliable tool. The index k and the decimal exponent $3k + 3$ are different numbers.

Powers of ten in measurement

An SI prefix multiplies a unit by an exact decimal factor. Symbols are case-sensitive: m is milli, M is mega. The official list includes ronna, quetta, ronto, and quecto, adopted in 2022 [4].

Large prefix	Symbol	Factor	Small prefix	Symbol	Factor
deca	da	10 ¹	deci	d	10 ⁻¹
hecto	h	10 ²	centi	c	10 ⁻²
kilo	k	10 ³	milli	m	10 ⁻³
mega	M	10 ⁶	micro	μ	10 ⁻⁶
giga	G	10 ⁹	nano	n	10 ⁻⁹
tera	T	10 ¹²	pico	p	10 ⁻¹²
peta	P	10 ¹⁵	femto	f	10 ⁻¹⁵
exa	E	10 ¹⁸	atto	a	10 ⁻¹⁸
zetta	Z	10 ²¹	zepto	z	10 ⁻²¹
yotta	Y	10 ²⁴	yocto	y	10 ⁻²⁴
ronna	R	10 ²⁷	ronto	r	10 ⁻²⁷
quetta	Q	10 ³⁰	quecto	q	10 ⁻³⁰

EXAMPLE 20 | Convert micrometres to metres

$7.5 \mu\text{m} = 7.5 \times 10^{-6} \text{ m} = 0.0000075 \text{ m}$.

To convert to nanometres, divide by 10^{-9} m per nm:

$7.5 \times 10^{-6 - (-9)} \text{ nm} = 7.5 \times 10^3 \text{ nm} = 7,500 \text{ nm}$.

SQUARE AND CUBE THE WHOLE UNIT

$1 \text{ cm}^2 = (10^{-2} \text{ m})^2 = 10^{-4} \text{ m}^2$.

$1 \text{ cm}^3 = (10^{-2} \text{ m})^3 = 10^{-6} \text{ m}^3$.

The prefix factor is raised to the same power as the unit.

A prefix changes the numerical size of the unit, not the underlying quantity. The same length can be written as 1 m, 100 cm, or 1,000 mm. A pure number such as 10^{79} has no measurement unit unless one is supplied.

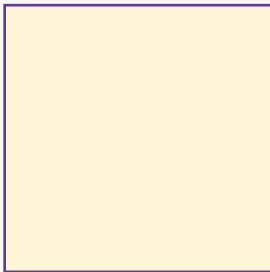
Digital storage distinction: a decimal kilobyte is 1,000 bytes; a kibibyte is 1,024 bytes. The IEC binary prefix kibi is not an SI decimal prefix.

Geometry multiplies the exponent

If a figure is enlarged by a factor of 10 in every linear dimension, lengths grow by 10, areas by 10², and volumes by 10³. Similarity is essential: stretching only one dimension gives a different result.



s



10s

SIDE ×10

AREA ×100

VOLUME ×1,000

Similar shapes: multiply the exponent by the dimension.

Quantity	Formula	If a length scales by 10 ⁿ
Square perimeter	$P = 4s$	P scales by 10 ⁿ
Square area	$A = s^2$	A scales by 10 ²ⁿ
Cube volume	$V = s^3$	V scales by 10 ³ⁿ
Circle area	$A = \pi r^2$	A scales by 10 ²ⁿ
Sphere volume	$V = (4/3)\pi r^3$	V scales by 10 ³ⁿ

EXAMPLE 21 | A microscopic cube

A cube has side $s = 10^{-3} \text{ m} = 1 \text{ mm}$.

Face area: $s^2 = (10^{-3})^2 \text{ m}^2 = 10^{-6} \text{ m}^2$.

Volume: $s^3 = (10^{-3})^3 \text{ m}^3 = 10^{-9} \text{ m}^3$.

The area is also 1 mm², and the volume is 1 mm³.

EXAMPLE 22 | Recover a length from a volume

A cube has volume 10¹² cm³. Its side is the cube root:

$s = (10^{12})^{1/3} \text{ cm} = 10^4 \text{ cm} = 100 \text{ m}$.

Check: $(100 \text{ m})^3 = 10^6 \text{ m}^3 = 10^{12} \text{ cm}^3$.

SURFACE AREA TO VOLUME

For a cube, $A/V = 6s^2/s^3 = 6/s$. Enlarging its side by 10 reduces this ratio by 10. Scale changes shape-related ratios even when the figure remains geometrically similar.

Counting, time, and physical constants

A useful scale calculation begins with a definition of what is counted and which units are used. Powers of ten simplify the arithmetic; they do not supply missing assumptions.

Quantity or convention	Value	Status
Second	1 s	Chosen time unit
Minute	60 s	Exact unit conversion
Hour	3,600 s	Exact unit conversion
24-hour day	86,400 s	Exact duration convention
364-day classroom year	31,449,600 s	Defined for this example
Speed of light in vacuum	299,792,458 m/s	Exact SI defining value [3]
Avogadro constant	$6.02214076 \times 10^{23} \text{ mol}^{-1}$	Exact SI defining value [3]

EXAMPLE 23 | A million and a billion seconds

Using a 24-hour day:

$$10^6 / 86,400 \approx 11.574 \text{ days.}$$

$$10^9 / 86,400 \approx 11,574.074 \text{ days.}$$

Using the explicitly chosen 364-day year:

$$10^9 / 31,449,600 \approx 31.797 \text{ years.}$$

A billion seconds is 1,000 times a million seconds.

A mole counts specified entities

One mole contains exactly $6.02214076 \times 10^{23}$ specified entities. Those entities could be atoms, molecules, ions, or other specified particles. One mole of water molecules contains two moles of hydrogen atoms and one mole of oxygen atoms because each water molecule is H_2O .

TIME MODELS MUST BE LABELED

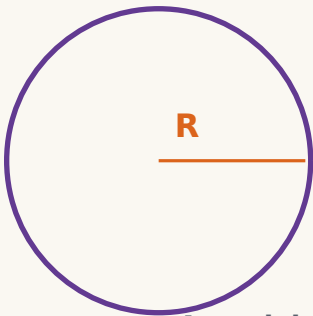
The 364-day year above is a classroom convention, not the Gregorian calendar year or the tropical year. It can be built from 13 equal months of 28 days, each containing four seven-day weeks. A 28-day month then contains 2,419,200 seconds.

Dimensional check: seconds $\div$ seconds per day = days. When a unit fails to cancel as intended, revisit the setup before trusting the numerical result.

How many atoms in the observable universe?

A common rough statement puts the ordinary-matter particle count near 10^{80} . Here is a transparent sphere model illustrating that scale. The inputs below are deliberately rounded; the calculation is a classroom estimate, not a precision cosmological fit. NASA describes an observable diameter of about 92 billion light-years [5].

Model input	Rounded assumption
Present observable radius R	4.4×10^{26} m
Mean ordinary-matter density ρ	4×10^{-28} kg/m ³
Representative nucleon mass m	1.67×10^{-27} kg



SPHERE MODEL

Volume: $(4/3)\pi R^3$

Mass: density $\times$ volume

Count: mass $\div$ proton mass

A model-based estimate, not a census of individual atoms.

1. Volume: $V = (4/3)\pi R^3 \approx 3.57 \times 10^{80}$ m³.
2. Ordinary-matter mass: $M = \rho V \approx 1.43 \times 10^{53}$ kg.
3. Nucleon-scale count: $M/m \approx 8.5 \times 10^{79}$, broadly 10^{80} .

The density is an illustrative assumption consistent with the broad ordinary-matter inventory, not the density of all cosmic energy. ESA's Planck summary gives ordinary matter about 4.9% of the total mass-energy inventory [6]. Dark matter and dark energy must not be counted as ordinary atoms.

WHAT THIS NUMBER DOES AND DOES NOT COUNT

Dividing by a nucleon mass estimates protons plus neutrons, approximately. A helium nucleus contains four nucleons, so nucleons and atoms are not interchangeable. Ionized plasma contains nuclei and free electrons rather than neutral atoms. Only the observable region is considered.

Consequently, “ 10^{79} atoms in the entire universe” is too definite. A safer statement is: ordinary-matter counts in the observable universe are often discussed on the enormous scale of roughly 10^{80} , with the exact meaning and estimate depending on the model.

Googol and googolplex

$$\text{googol} = 10^{100}$$

$$\text{googolplex} = 10^{(10^{100})}$$

A googol has 100 zeros and 101 digits. A googolplex has a googol zeros and a googol plus one digits. The exponent is doing the essential work: a googolplex is ten raised to an exponent of 10^{100} , rather than ten raised to 100×100 [2].

Expression	Simplified meaning	Comparison
$(10^{100})^2$	10^{200}	Square of a googol
$(10^{100})^{100}$	10^{10000}	A googol to the hundredth power
$10^{(10^{100})}$	A googolplex	Far larger than either row above
$10^{-(10^{100})}$	Reciprocal of a googolplex	Positive and smaller than 10^{-100}

EXAMPLE 24 | Compare by comparing exponents

Which is larger: a centillion or a googolplex?

Centillion = 10^{303} . Googolplex = $10^{(10^{100})}$.

Because $303 < 10^{100}$ and 10^x increases with x ,
 $10^{303} < 10^{(10^{100})}$.

A short definition can specify an immense integer

Writing the definition is very different from writing every decimal digit. The expression $10^{(10^{100})}$ fits on one line. Expanding it would require a googol zeros. Even a hypothetical inventory of 10^{80} objects, storing one digit per object, falls short by a factor of about 10^{20} .

STILL FINITE

A googolplex is an integer with a last digit, namely 0. Adding 1 makes a larger finite integer. The difficulty of physically writing all its digits does not make it infinite.

A googol is larger than a commonly quoted cosmic particle count, but mathematics places no requirement that each digit or object in a definition correspond to something physically present.

Power towers and Knuth arrows

Exponentiation is not associative. In general, $(a^b)^c$ differs from $a^{(b^c)}$. A power tower conventionally groups from the top downward: first evaluate its highest exponent. Parentheses make the intended order explicit.

$$10^3$$

$$1,000$$

$$(10^3)^2$$

$$10^6$$

$$10^{(3^2)}$$

$$10^9$$

$$10^{(10^3)}$$

$$10^{1000}$$

Parentheses decide which exponent is evaluated first.

Notation	Meaning	Small example
$a \uparrow b$	Ordinary exponentiation a^b	$3 \uparrow 3 = 27$
$a \uparrow \uparrow b$	A right-associated tower of b copies of a	$3 \uparrow \uparrow 2 = 3^3 = 27$
$a \uparrow \uparrow \uparrow b$	Iterated double-arrow operation	$3 \uparrow \uparrow \uparrow 2 = 3 \uparrow \uparrow 3$

Build a tower in steps

$$3 \uparrow \uparrow 3 = 3^{(3^3)} = 3^{27} = 7,625,597,484,987.$$

$$\text{But } (3^3)^3 = 3^9 = 19,683.$$

A third level in a tower raises 3 to the value of the two-level tower. It does not simply multiply the exponent by 3.

A RECURSIVE DEFINITION

For integers $a \geq 2$, set $a \uparrow^1 b = a^b$ and $a \uparrow^k 0 = 1$.

For $k \geq 2$ and $b \geq 1$:

$$a \uparrow^k b = a \uparrow^{k-1} (a \uparrow^k (b - 1)).$$

The superscript on $\uparrow$ tells how many arrows are used; it is not a power applied to the final answer.

For example, $3 \uparrow \uparrow \uparrow 3 = 3 \uparrow \uparrow (3 \uparrow \uparrow 3)$. It is a tower of 3s whose height is 7,625,597,484,987. Its compact description is exact even though writing its expansion is impractical.

The notation is associated with Donald Knuth. The definitions and simple expansions above are sufficient for the next page; the general hierarchy is optional enrichment [2, 7].

Finite giants beyond a googolplex

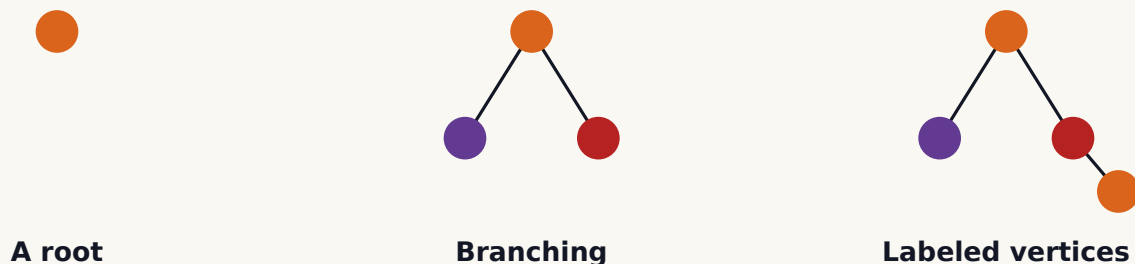
Graham's number

Define $g_1 = 3 \uparrow \uparrow \uparrow \uparrow 3$. For each next term, let $g_{n+1} = 3 \uparrow^{g_n} 3$: the previous value tells how many arrows to use. The conventional Graham's number is $G = g_{64}$. It is a famous finite upper bound associated with a problem in Ramsey theory, not the greatest number [7].

TREE(3)

Friedman's TREE function concerns sequences of finite rooted trees with a fixed number of labels, a size limit tied to each tree's position, and a prohibition on an earlier tree embedding into a later one in a specified structure-preserving way. TREE(3) is finite and vastly exceeds Graham's number [7, 8].

SCHEMATIC ROOTED TREES (NOT A TREE(3) SEQUENCE)



Busy Beaver: beyond a single growth recipe

For a fixed Turing-machine convention, a Busy Beaver value can be defined as the longest runtime among halting machines of a given size. Each such maximum is finite, but no algorithm computes the function for every input. This is a theorem about computability, not just a complaint that computers are slow [9].

NO FINITE CHAMPION CAN BE FINAL

If G is Graham's number, then $G + 1 > G$. If $T = \text{TREE}(3)$, then $T + 1 > T$. A new name is unnecessary: ordinary addition already constructs a larger integer. A rule-bound “largest describable number” depends on its chosen language and description-length limit.

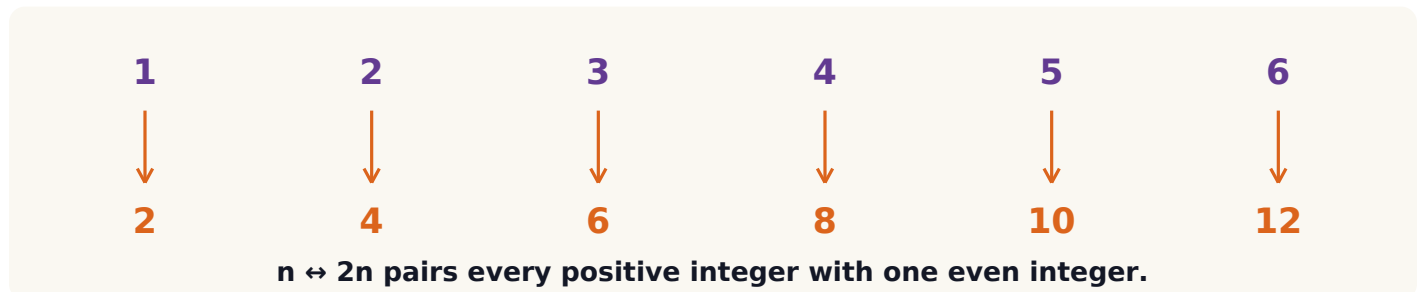
The sketches of trees illustrate roots and branching only. They are not a construction of a longest legal TREE sequence. This page introduces the ideas; proving their advanced bounds requires considerably more mathematics.

Infinity is not the last integer

The integers do not finish at a final number called infinity. The notation $n \rightarrow \infty$ means that n grows beyond every finite bound. Likewise, $10^n \rightarrow \infty$ describes unbounded growth; it does not make any individual finite power of ten infinite.

Statement	Interpretation
10^{100} is finite	It is a specific integer, with 101 digits.
$10^{-n} \rightarrow 0$	The positive terms approach zero as n increases.
$10^n \rightarrow \infty$	The values eventually exceed every finite bound.
$1/0$	Undefined in ordinary real arithmetic.
$\infty - \infty$	No single value follows from this symbol pattern.

Counting infinite collections



A set is countably infinite when its members can be paired one-to-one with the positive integers. The even positive integers are countably infinite: pair n with $2n$. Having every even integer in the pairing does not require a last pair.

The real numbers in the interval $(0, 1)$ cannot be listed this way. Cantor's diagonal method constructs a decimal that differs from the n th listed decimal at its n th place. Choosing only digits 1 and 2 avoids the alternate-decimal problem involving strings of 9s.

DIFFERENT SIZES OF INFINITE SETS

The cardinality of the positive integers is written $\aleph_0$. The real numbers have a strictly larger cardinality. Neither cardinal is the greatest possible cardinal: the set of all subsets of a set has greater cardinality than the original set.

There is no greatest real number, no greatest integer, and no greatest cardinal. Extended-number systems and set theory each have precise rules; treating ∞ as an ordinary real number creates errors.

Review: habits that prevent errors

Tempting error	Correct statement	Why
$10^3 = 10 \times 3$	$10^3 = 1,000$	An exponent counts factors.
$10^0 = 0$	$10^0 = 1$	Continue the divide-by-ten pattern.
$10^{-3} = -1,000$	$10^{-3} = 0.001$	A negative exponent means reciprocal.
$10^3 + 10^2 = 10^5$	$10^3 + 10^2 = 1,100$	Addition does not add exponents.
$(10^3)^2 = 10^9$	$(10^3)^2 = 10^6$	Multiply these exponents.
10^{79} has 79 digits	10^{79} has 80 digits	Include the leading 1.
$1 \text{ cm}^2 = 0.01 \text{ m}^2$	$1 \text{ cm}^2 = 0.0001 \text{ m}^2$	Square the conversion factor.
Googolplex = 10^{200}	Googolplex = $10^{(10^{100})}$	The googol is the exponent.

A four-step checking routine

1. Identify the object: exact number, measurement, estimate, or limit.
2. Identify the operation: addition, multiplication, division, root, or tower.
3. Apply the matching rule and keep units attached.
4. Check the sign, approximate scale, and notation before naming the answer.

ESSENTIAL FORMULAS

$$10^a 10^b = 10^{a+b}; \quad 10^a / 10^b = 10^{a-b}; \quad (10^a)^b = 10^{ab}.$$

$$10^{-n} = 1/10^n; \quad \log_{10}(10^x) = x.$$

$$\text{Digits of } 10^n = n + 1 \text{ for integer } n \geq 0.$$

$$\text{Short-scale kth -illion} = 10^{3k+3} \text{ for integer } k \geq 1.$$

Explain a result in more than one form whenever practical. For example, 10^{-6} , $1/1,000,000$, 0.000001 , and one millionth all describe the same number. Recognizing that agreement is a stronger check than memorizing a single spelling.

Readiness target: complete the routine calculations without a calculator. Leave giant values in exact power notation. Decimal approximations are requested explicitly when useful.

Exercises 01-12: Place value and names

Show the rule or conversion behind each answer. Use the page references in the contents to review unfamiliar concepts. Space below each item is for a compact answer; longer reasoning may continue on separate paper.

01. Write 10^5 in ordinary decimal notation.

02. Write 10^{-4} as a fraction and as a decimal.

03. Evaluate 10^0 and explain why it is not zero.

04. How many zeros and how many digits are in 10^{37} ?

05. Write one billion as a power of ten in the short scale.

06. Name 10^{75} in the short scale.

07. Name 10^{79} in the short scale.

08. Name 10^{80} in the short scale.

09. How many zeros occur between the decimal point and the 1 in 10^{-9} ?

10. Explain why 0.000001 is not the smallest positive real number.

11. Compare 10^{-3} and -10^3 using $<$ or $>$.

12. Simplify $\sqrt{(10^6)}$ exactly.

Check your work with pages 34-36 after completing the set. For comparison questions, explain whether you are comparing signed values, magnitudes, or ratios.

Exercises 13-24: Exponent laws and notation

Show the rule or conversion behind each answer. Use the page references in the contents to review unfamiliar concepts. Space below each item is for a compact answer; longer reasoning may continue on separate paper.

13. Simplify $10^8 \times 10^5$.

14. Simplify $10^4/10^9$.

15. Simplify $(10^3)^4$.

16. Simplify $(10^{-2})^3$.

17. Simplify $(10^7 \times 10^{-4})/10^2$.

18. Write 72,000,000 in scientific notation.

19. Write 0.0000034 in scientific notation.

20. Normalize 45×10^8 .

21. Multiply $(2 \times 10^6)(6 \times 10^3)$.

22. Divide $(9 \times 10^7)/(3 \times 10^2)$.

23. Add $4.2 \times 10^5 + 3 \times 10^4$.

24. Subtract $8 \times 10^6 - 5 \times 10^5$.

Check your work with pages 34-36 after completing the set. For comparison questions, explain whether you are comparing signed values, magnitudes, or ratios.

Exercises 25-36: Comparison, units, and geometry

Show the rule or conversion behind each answer. Use the page references in the contents to review unfamiliar concepts. Space below each item is for a compact answer; longer reasoning may continue on separate paper.

25. How many times larger is 10^{79} than 10^{78} ?

26. How many times larger is 10^{79} than 10^{75} ?

27. Which is greater: 9×10^{12} or 1.1×10^{13} ?

28. How many digits are in the integer 6×10^{23} ?

29. Evaluate $\log_{10}(10^{-12})$.

30. Convert $3.2 \mu\text{m}$ to metres and to nanometres.

31. Convert 1 cm^2 to square metres.

32. A cube has side 10^2 cm . Find its volume in cm^3 .

33. A similar cube has ten times that side. By what factor does its volume grow?

34. A cube has volume 10^{18} m^3 . Find its side length.

35. Find the seconds in a 28-day month using 24-hour days.

36. Use 364-day years. Give 10^9 seconds in years to three decimal places.

Check your work with pages 34-36 after completing the set. For comparison questions, explain whether you are comparing signed values, magnitudes, or ratios.

Exercises 37-48: Large numbers and interpretation

Show the rule or conversion behind each answer. Use the page references in the contents to review unfamiliar concepts. Space below each item is for a compact answer; longer reasoning may continue on separate paper.

37. What is the short-scale power for the 25th -illion?

38. How many digits are in one short-scale centillion?

39. Write the square of a googol as a power of ten.

40. State the exponent in the definition of a googolplex.

41. Compare $(10^2)^3$ with $10^{(2^3)}$.

42. Evaluate $2 \uparrow \uparrow 3$.

43. Evaluate $3 \uparrow \uparrow 2$.

44. If G is Graham's number, give a larger finite integer.

45. Is $\text{TREE}(3)$ finite or infinite?

46. Does 10^{-n} ever equal zero at a finite positive integer n ?

47. Using $V = 3.57 \times 10^{80} \text{ m}^3$ and $\rho = 4 \times 10^{-28} \text{ kg/m}^3$, estimate M .

48. Why does dividing cosmic ordinary-matter mass by a proton mass not give an exact neutral-atom census?

Check your work with pages 34-36 after completing the set. For comparison questions, explain whether you are comparing signed values, magnitudes, or ratios.

Worked answers 01-16

Compare the method as well as the result. Exact expressions are preferred unless an approximation was requested.

01. 100,000: five zeros follow the 1.

02. $1/10,000 = 0.0001$.

03. 1. For example, $10^3/10^3 = 10^0 = 1$.

04. 37 zeros and 38 digits.

05. 10^9 .

06. One quattuorvigintillion.

07. Ten quinvigintillion.

08. One hundred quinvigintillion.

09. Eight. The 1 is in decimal place 9.

10. Divide it by 10: 0.0000001 is positive and smaller.

11. $10^{-3} > -10^3$ because $0.001 > -1,000$.

12. $10^3 = 1,000$; the principal square root is positive.

13. $10^{8+5} = 10^{13}$.

14. $10^{4-9} = 10^{-5}$.

15. $10^{3 \times 4} = 10^{12}$.

16. 10^{-6} .

SELF-CHECK

For every integer power of ten, check the direction of the exponent and count the leading 1 in the total digits. Number names follow the short scale throughout this answer key.

Worked answers 17-32

Compare the method as well as the result. Exact expressions are preferred unless an approximation was requested.

17. $10^{7-4-2} = 10^1 = 10$.

18. 7.2×10^7 .

19. 3.4×10^{-6} .

20. 4.5×10^9 .

21. $12 \times 10^9 = 1.2 \times 10^{10}$.

22. 3×10^5 .

23. $(4.2 + 0.3) \times 10^5 = 4.5 \times 10^5$.

24. $(8 - 0.5) \times 10^6 = 7.5 \times 10^6$.

25. $10^{79-78} = 10$ times.

26. $10^4 = 10,000$ times.

27. 1.1×10^{13} , because $9 \times 10^{12} = 0.9 \times 10^{13}$.

28. 24 digits: it lies between 10^{23} and 10^{24} .

29. -12.

30. 3.2×10^{-6} m; 3.2×10^3 nm = 3,200 nm.

31. $(10^{-2} \text{ m})^2 = 10^{-4} \text{ m}^2$.

32. $(10^2)^3 = 10^6 \text{ cm}^3$.

SELF-CHECK

Align powers before adding or subtracting. In products, operate on coefficients and exponents separately, then normalize. Keep unit powers attached until the last step.

Worked answers 33-48

Compare the method as well as the result. Exact expressions are preferred unless an approximation was requested.

33. $10^3 = 1,000$.

34. $(10^{18})^{1/3} = 10^6 \text{ m}$.

35. $28 \times 24 \times 60 \times 60 = 2,419,200 \text{ s}$.

36. $10^9/31,449,600 \approx 31.797 \text{ years}$.

37. $10^{3(25)+3} = 10^{78}$, quinvigintillion.

38. 10^{303} has 304 digits.

39. $(10^{100})^2 = 10^{200}$.

40. The exponent is 10^{100} , itself a googol.

41. $(10^2)^3 = 10^6$; $10^{(2^3)} = 10^8$. The second is 100 times the first.

42. $2^{(2^2)} = 2^4 = 16$.

43. $3^3 = 27$.

44. $G + 1$ is one example.

45. Finite. Its enormous size does not change that fact.

46. No. It is $1/10^n$, which is positive. Its limit is zero.

47. $M = \rho V = 14.28 \times 10^{52} \approx 1.43 \times 10^{53} \text{ kg}$.

48. Atoms contain different numbers of nucleons; cosmic matter can be ionized; the inputs are estimates. It approximates a nucleon-scale count.

SELF-CHECK

A finite definition can name an immense integer. A sequence can have a finite limit without ever reaching it. Physical estimates require a model and cannot be substituted for exact mathematical identities.

Glossary of essential ideas

Term	Meaning in this book
Base	The quantity repeatedly multiplied in a positive integer power; 10 in 10 ^N .
Coefficient	The factor a in $a \times 10^n$.
Decimal exponent	An exponent applied to base 10.
Digit	One of 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 in decimal notation.
Exponent	The index that specifies a power; extended through reciprocals, roots, and real exponentiation.
Finite	A specific bounded size or value, however large.
Googol	The exact integer 10 ¹⁰⁰ .
Googolplex	The exact integer 10 ^(10¹⁰⁰) .
Infimum	The greatest lower bound of a set. For the positive reals, it is 0.
Integer	A whole-number value, positive, negative, or zero.
Logarithm	The exponent needed to obtain a positive value from a chosen base.
Magnitude	Size; for a real number, often its absolute value.
Nucleon	A proton or neutron in the ordinary nuclear-matter count used here.
Power tower	Repeated exponentiation with a specified grouping, usually from the top down.
Reciprocal	1/x for $x \neq 0$.
Scientific notation	$a \times 10^n$ with integer n and $1 \leq a < 10$.
Short scale	The naming system in which the kth -illion is 10 ^{3k+3} .
Significant digits	Digits used to express the reported precision of a measured value.
Tetration	Repeated exponentiation, represented here by a double arrow.
Unbounded	Exceeding every proposed bound within the specified direction or context.

A glossary gives working meanings. Some words have broader technical definitions in advanced mathematics or in other scientific disciplines.

Sources and reading notes

These sources support conventions, physical values, and advanced context. Mathematical identities can also be checked directly from the examples. URLs are clickable. Web references checked for this edition on October 1, 2026.

[1] OpenStax. College Algebra 2e, §1.2, “Exponents and Scientific Notation.”

<https://openstax.org/books/college-algebra-2e/pages/1-2-exponents-and-scientific-notation>

Foundational exponent and scientific-notation rules. The lessons and exercises here are original.

[2] Tom Davis. Big Numbers. Math-circle notes.

<https://www.geometer.org/mathcircles/bignumbers.pdf>

Background on large-number names, googol notation, and growth. Extended spellings are conventional; the powers are the exact identifiers.

[3] NIST. CODATA recommended values of the fundamental physical constants.

<https://physics.nist.gov/cuu/pdf/all.pdf>

Physical constants, including the Planck length, speed of light, proton mass, and Avogadro constant. Values in worked science estimates are deliberately rounded.

[4] BIPM. SI prefixes; Resolution 3 of the 27th CGPM (2022).

<https://www.bipm.org/en/measurement-units/si-prefixes>

Authoritative SI prefix names, symbols, and multiplying factors.

[5] NASA. “How Big is Space? We Asked a NASA Expert,” Episode 61.

<https://www.nasa.gov/science-research/astrophysics/how-big-is-space-we-asked-a-nasa-expert-episode-61/>

Observable-universe size context. Page 24 develops its own simplified calculation.

[6] ESA. “Planck reveals an almost perfect Universe.”

https://www.esa.int/Science_Exploration/Space_Science/Planck/Planck_reveals_an_almost_perfect_Universe

Ordinary matter as roughly 4.9% of the cosmic mass-energy inventory.

READING PHYSICAL NUMBERS

An exact SI defining constant, a measured physical quantity, a model assumption, and an order-of-magnitude estimate carry different kinds of certainty. The universe calculation labels its rounded inputs so that changing an assumption changes the result transparently.

Advanced sources and subject index

[7] Harvey Friedman. FOM post, " $n(3) < \text{Graham's number} < n(4) < \text{TREE}[3]$," 29 March 2006.

<https://fomarchive.ugent.be/2006-March/010290.html>

Comparison of finite giants; discussion of the conventional Graham-number construction.

[8] Harvey Friedman. FOM post 272, "Sigma01/optimal," 24 March 2006.

<https://fomarchive.ugent.be/2006-March/010260.html>

Original discussion of size-controlled sequences of labeled finite trees and embedding restrictions.

[9] Scott Aaronson. The Busy Beaver Frontier.

<https://www.scottaaronson.com/papers/bb.pdf>

Computability and Busy Beaver functions. The textbook avoids claims about a latest numerical record.

Subject index

Topic	Pages	Topic	Pages
Atoms / universe estimate	19, 24	Infinitesimals	8
Avogadro constant	23	Infinity / cardinality	28
Busy Beaver	27	Knuth arrows	26-27
Centillion	13, 20, 25	Logarithms	12
Comma groups	19	Negative exponents	5, 7-9
Decimal place value	4-5	Number names	13-20
Digit count	4, 12, 14-20	Planck length	8
Exercises / answers	30-36	Power towers	26
Exponent laws	9, 29	Quinvigintillion	13, 17, 19
Fractional exponents	6	Scientific notation	10-12
Geometry / scaling	22	Short and long scales	13
Googol / googolplex	18, 20, 25	SI prefixes	21
Graham's number	27	Time conversions	23
Greatest / smallest	3, 8, 28	TREE(3)	27
Zero exponent	5	Zeros in a power	4, 7, 19

The indexed route is a starting point, never an endpoint: once you can define a finite number, you can construct a larger one; once you have a positive real number, you can construct a smaller positive one.