

FRACTIONAL EXPONENTS

$$1^{1/2} = 1$$

A numerical journey through roots, powers, real numbers, and the complex plane.

THE COMPLETE NUMBER ATLAS

21 integer bases, from -10 through +10.

100 ordered numerator-denominator pairs for every base.

2,100 entries, with equivalent fractions kept visible.

Learn the rule. Work the example. Read the pattern.

BASE	FRACTIONAL POWER	MEANING
4	$4^{1/2} = 2$	A square root
8	$8^{2/3} = 4$	A cube root, then a square
-8	$(-8)^{1/3} = -2$	The real cube-root convention
-1	$(-1)^{1/2} = i$	The principal complex value

INSIDE THIS BOOK

Definitions and worked examples • exact and approximate values • negative-base rules • principal complex powers • classroom exercises and solutions • complete numerical reference tables.

Positive-base chapters progress from 1 to 10. Each magnitude includes its negative companion; zero has a separate reference section.

Contents & reading guide

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ATLAS CHAPTER	NEGATIVE BASE	POSITIVE BASE
Zero: all 100 entries	13-14	13-14
Magnitude 1	15-17	18-19
Magnitude 2	20-22	23-24
Magnitude 3	25-27	28-29
Magnitude 4	30-32	33-34
Magnitude 5	35-37	38-39
Magnitude 6	40-42	43-44
Magnitude 7	45-47	48-49
Magnitude 8	50-52	53-54
Magnitude 9	55-57	58-59
Magnitude 10	60-62	63-64

Reading key: “=” marks an exact displayed value; “ $\approx$ ” marks a value rounded to six decimal places. “No real value” identifies an expression outside the real domain. The principal complex column uses $\text{Arg}(-a) = \pi$.

Begin with one

$$1^{1/2} = 1$$

The exponent $1/2$ asks for a square root. Because $1 \times 1 = 1$, the nonnegative square root of one is one. This small calculation contains the central idea: a fractional exponent links a root operation with a power operation.

Question: what changes when the denominator changes?

$$1^{1/3} = 1^{1/4} = 1^{1/10} = 1$$

Nothing changes in the result. Multiplying any positive number of copies of one still gives one. Every positive integer root of one has the principal real value one. Raising that result to any positive integer power also gives one.

The general positive-base rule

$$a^{p/q} = (\text{qth root of } a)^p, a > 0$$

Read the denominator q as the root index and the numerator p as the power. For positive bases, taking the root first or the power first gives the same positive result. A rational exponent is a number, so equivalent fractions represent the same exponent. [1]

TERM	STEP 1: ROOT	STEP 2: POWER
$4^{3/2}$	Square root of 4 = 2	$2^3 = 8$
$8^{2/3}$	Cube root of 8 = 2	$2^2 = 4$
$9^{3/2}$	Square root of 9 = 3	$3^3 = 27$
$1^{10/9}$	Ninth root of 1 = 1	$1^1 = 1$

ONE EXPRESSION IS NOT AN ENTIRE EQUATION

The expression $4^{1/2}$ has the principal real value 2. The equation $x^2 = 4$ has two solutions, $x = 2$ and $x = -2$. The symbol for a principal root selects one value; solving an equation asks for every value that works.

Learning goals: translate between roots and exponents; reduce fractions; identify real domains; compare exact values; and recognize when a complex convention is being used.

The exponent is a rational number

WORD	PRECISE MEANING
Base	The number being raised to a power.
Exponent	The number that specifies the power operation.
Numerator p	The top integer in the displayed fraction p/q.
Denominator q	The bottom integer; here it is always positive.
Lowest terms r/s	The fraction after dividing p and q by gcd(p,q).
Root index	The positive integer that specifies which root is taken.

All combinations are retained

For each base, p and q independently take every integer from 1 through 10. Thus there are $10 \times 10 = 100$ ordered fraction forms. They represent 63 distinct rational numbers. Repeated values teach equivalence; they are not missing or duplicate records by accident.

EQUIVALENT FRACTION FORMS	LOWEST TERMS
1/2, 2/4, 3/6, 4/8, 5/10	1/2
1/3, 2/6, 3/9	1/3
2/3, 4/6, 6/9	2/3
1/1, 2/2, 3/3, ..., 10/10	1

How the atlas is ordered

Numerators progress from 1 to 10; denominators generally progress from 1 to 10 within each numerator. The opening block is 1/2 through 1/10, followed by 1/1. In the final numerator block, 10/10 appears immediately before 10/9. Every pair still appears exactly once.

BOUNDARIES OF THIS ATLAS

The smallest exponent is 1/10 and the largest is 10/1. There are 45 displayed pairs below 1, 10 equal to 1, and 45 above 1. Denominator 1 and equal-numerator-and-denominator cases are included. Zero and negative exponents are not part of these tables.

A proper fraction has numerator smaller than denominator. An improper fraction has numerator at least as large as denominator. Fractional notation can therefore represent a whole number: $6/3 = 2$.

Positive bases: roots, then powers

Example A: an exact integer

$8^{4/6} = 8^{2/3}$. First reduce $4/6$ to $2/3$. Take the cube root of 8 to get 2. Square 2 to get 4. The displayed fraction changes, but the exponent and value do not.

$$8^{4/6} = (\text{cube root of } 8)^2 = 2^2 = 4$$

Example B: an exact radical and a decimal

$2^{3/2} = (\text{square root of } 2)^3 = 2\sqrt{2}$. This radical is exact. Its decimal expansion does not terminate or repeat, so a short decimal must be marked as an approximation.

$$2^{3/2} = 2\sqrt{2} \approx 2.828427$$

Example C: split an improper exponent

$$10^{10/9} = 10^{1 + 1/9} = 10 \times 10^{1/9}$$

The value is a little larger than ten because $10^{1/9}$ is a little larger than one. Splitting a fraction into a whole number and a remainder often makes its size easier to predict.

BASE $a > 1$	SIZE OF THE RESULT	EXAMPLE
$0 < r < 1$	$1 < a^r < a$	$9^{1/2} = 3$
$r = 1$	$a^r = a$	$9^{4/4} = 9$
$r > 1$	$a^r > a$	$9^{3/2} = 27$

WHEN IS THE RESULT RATIONAL?

For an integer $a > 0$ and a reduced exponent r/s , the result is rational exactly when a is a perfect s th power of an integer. Equivalently, every prime-factor exponent of a is divisible by s . In that case the result is an integer. For example, $4 = 2^2$ and $9 = 3^2$ permit half-integer exponents; $8 = 2^3$ permits exponents with reduced denominator 3.

Why? In a prime factorization, raising a to r/s multiplies each prime exponent by r/s . Since r and s are coprime, those exponents are integers precisely when s divides each original prime exponent.

Negative bases: reduce first

For a negative base, the real-number convention starts with the exponent in lowest terms. Write $p/q = r/s$ with $\gcd(r,s) = 1$. The parity of s decides whether a real value exists. [2]

REDUCED DENOMINATOR	REAL-NUMBER RESULT
s odd	A real value exists: $(-a)^{r/s} = (-1)^r a^{r/s}$.
s even	No real value exists for $a > 0$.

Example A: the displayed denominator is even

$$(-8)^{2/6} = (-8)^{1/3} = -2$$

The fraction $2/6$ reduces to $1/3$, whose denominator is odd. The real cube root of -8 is -2 . The sign stays negative because the reduced numerator is odd.

Example B: the reduced numerator is even

$$(-8)^{4/6} = (-8)^{2/3} = (-2)^2 = 4$$

This time the reduced numerator is even, so the real result is positive. A negative base does not guarantee a negative result.

Example C: an even denominator remains

$$(-4)^{2/4} = (-4)^{1/2}$$

There is no real value: the square of a real number cannot be negative. In the principal complex convention the value is $2i$. The table therefore shows “No real value” and gives a separate complex result.

DO NOT REPLACE A RATIONAL POWER BLINDLY

For a negative base, unreduced “power first, then root” can change the result. The real rational power $(-8)^{2/6}$ is -2 , but the nonnegative sixth root of $(-8)^2$ is $+2$. Reduce the exponent before choosing a real root.

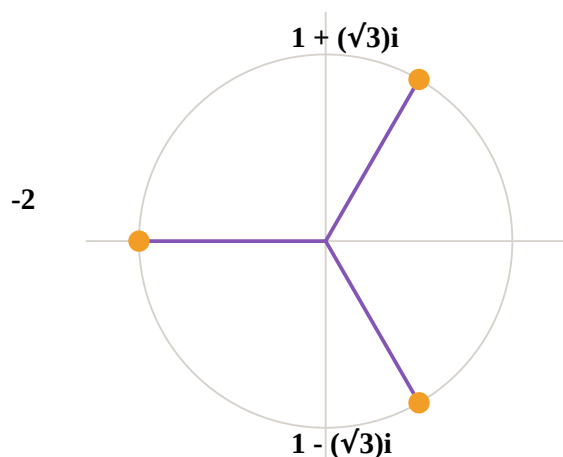
Parentheses matter: $(-4)^2 = 16$, whereas $-4^2 = -(4^2) = -16$. In every negative-base atlas entry, parentheses make the base explicit. A negative base is also different from a negative exponent.

Complex powers & root geometry

A complex number has the form $x + yi$, where $i^2 = -1$. The atlas uses the principal argument $\text{Arg}(-a) = \pi$ for $a > 0$. Its principal complex power is defined through that argument. [3]

$$P(-a, r/s) = a^{r/s} [\cos(\pi r/s) + i \sin(\pi r/s)]$$

Here P labels the principal complex value. The magnitude is $a^{r/s}$; the angle is $\pi r/s$, interpreted modulo 2π . This choice can differ from the real odd-root convention. For example, $(-8)^{1/3}$ has real value -2 but principal complex value $1 + (\sqrt{3})i$.



The three cube roots of -8

Each point has magnitude 2.

Each point cubes to -8.

The point at 60° is the principal complex value.

The point on the negative real axis is the real cube root.

All branches: a complete exact description

$$z_k = a^{r/s} [\cos((r\pi + 2\pi k)/s) + i \sin((r\pi + 2\pi k)/s)], k = 0, \dots, s - 1$$

For coprime positive integers r and s , these are the s distinct solutions of $z^s = (-a)^r$. The $k = 0$ term is the atlas principal value. If s is odd, one branch gives the real convention; if s is even, none is real.

A POWER AND A SOLUTION SET

For a positive base, the atlas selects its positive real power. The equation $z^s = a^r$ also has s complex roots when $a > 0$; only one is the selected positive value. For a negative base, the real and principal complex columns name two conventions rather than listing every branch.

Exponent laws & their boundaries

Use the familiar exponent laws freely with positive real bases. With negative or complex bases, state the domain and the selected root or logarithm branch before applying a transformation. [1, 3]

LAW FOR $a, b > 0$	EXAMPLE
$a^r a^s = a^{r+s}$	$4^{1/2} \times 4^{3/2} = 4^2 = 16$
$a^r / a^s = a^{r-s}$	$9^{3/2} / 9^{1/2} = 9$
$(a^r)^s = a^{rs}$	$(8^{2/3})^{3/2} = 8$
$(ab)^r = a^r b^r$	$(4 \times 9)^{1/2} = 2 \times 3 = 6$
$(a/b)^r = a^r / b^r$	$(9/4)^{1/2} = 3/2$
$a^{-r} = 1/a^r$	$4^{-1/2} = 1/2$

A real-domain counterexample

$$((-1)^2)^{1/2} = 1, \text{ but } (-1)^{2 \times 1/2} = -1$$

This is why “power of a power” needs conditions. Squaring loses the sign, and the principal square root does not recover it. The correct identity for any real x is $\sqrt{x^2} = |x|$, not simply x .

A complex-domain counterexample

$$(-1)^{1/2} (-1)^{1/2} = i^2 = -1$$

$$((-1)(-1))^{1/2} = 1^{1/2} = 1$$

The product-to-a-power identity fails here when principal square roots are selected independently. The two expressions take different routes through the branch choice. This does not contradict the positive-base rule; its assumptions are absent.

FOUR CHECKS BEFORE SIMPLIFYING

1. Put parentheses around a negative base.
2. Reduce every rational exponent.
3. Decide whether the task uses real values or principal complex values.
4. Check that each exponent law is valid on that domain.

Approximation & practical meaning

Estimate a root by bracketing

To estimate $2^{1/2}$, compare squares: $1.4^2 = 1.96$ and $1.5^2 = 2.25$. Therefore the root lies between 1.4 and 1.5. Refine the interval: $1.41^2 = 1.9881$ and $1.42^2 = 2.0164$. Continuing gives more digits without changing the exact quantity.

Improve the estimate with Newton's method

$$x_{\text{next}} = ((s - 1)x + A/x^{s-1})/s$$

For the positive sth root of $A > 0$, choose $x > 0$. For $A = 2$ and $s = 2$, start at $x = 1.5$. The next estimates are 1.416666667 and 1.414215686. Root first, then raise to the reduced numerator. Keep guard digits until the final rounding.

APPLICATION	MODEL	EXAMPLE
Square side from area	$L = A^{1/2}$	$A = 9 \text{ m}^2$ gives $L = 3 \text{ m}$.
Cube edge from volume	$L = V^{1/3}$	$V = 8 \text{ m}^3$ gives $L = 2 \text{ m}$.
Similar-solid surface scale	Surface factor $= k^{2/3}$	Volume $\times 8$ gives surface $\times 4$.
Equal growth multiplier	$g = (F/I)^{1/n}$	Doubling in 3 periods: $g = 2^{1/3}$.

Why roots appear in measurement

Length, area, and volume scale with different integer powers. Recovering a length reverses those powers, introducing fractions in the exponent. A square root reverses a square; a cube root reverses a cube. In growth models, an nth root finds the equal multiplier that repeats n times to produce an overall factor.

READ THE DECIMALS CORRECTLY

Atlas decimals are rounded to six places after the decimal point. A displayed “=” is exact; a displayed “ $\approx$ ” is approximate. Complex values round their real and imaginary components separately. Use the exact defining expression, not a rounded table entry, for further symbolic work.

Units participate in the power: $(\text{m}^2)^{1/2} = \text{m}$ and $(\text{m}^3)^{1/3} = \text{m}$. A growth factor is dimensionless. A model is meaningful only when its quantities and units match its assumptions.

Practice: 30 questions

Work in the real-number convention unless a question explicitly says “principal.” Give exact values where possible. Use a separate sheet for calculations; the atlas can be used to check your answers.

1. Evaluate $1^{7/10}$.
2. Evaluate $4^{1/2}$.
3. Evaluate $8^{1/3}$.
4. Evaluate $8^{2/3}$.
5. Evaluate $9^{3/2}$.
6. Evaluate $4^{5/2}$.
7. Reduce $6/8$; rewrite $2^{6/8}$.
8. List every atlas form equal to $1/2$.
9. Which is larger: $3^{1/2}$ or $3^{2/3}$?
10. Is $5^{7/10}$ between 1 and 5?
11. Give the real value of $(-8)^{1/3}$.
12. Give the real value of $(-8)^{2/3}$.
13. Give the real value of $(-8)^{2/6}$.
14. Give the real value of $(-8)^{4/6}$.
15. Does $(-9)^{3/6}$ have a real value?
16. Give the principal value of $(-9)^{1/2}$.
17. Give the principal value of $(-4)^{3/2}$.
18. Give the real value of $(-1)^{4/5}$.
19. Give the principal value of $(-1)^{4/5}$.
20. Evaluate $0^{10/9}$.
21. Simplify $4^{1/2} \times 4^{3/2}$.
22. Simplify $9^{3/2}/9^{1/2}$.
23. Explain why $2^{1/2}$ is irrational.
24. Find the side of a square of area 10 m^2 .
25. Find the edge of a cube of volume 8 m^3 .
26. Volume increases by a factor of 8 for similar solids. Surface factor?
27. Find the equal multiplier for doubling in 3 periods.
28. Compare $(-4)^2$ and -4^2 .
29. How many fraction forms and distinct exponents occur?
30. Rewrite $10^{10/9}$ using $1 + 1/9$.

EXTENSION INVESTIGATION

Choose a negative base. Mark every row whose real value exists, then compare those rows with the principal complex column. Find one exponent for which the two values agree and one for which they differ. Explain the difference using the reduced denominator and the selected angle.

Answer key with reasoning

For irrational values, a radical or exact fractional power is preferable to an unmarked rounded decimal. In complex answers, i denotes the imaginary unit.

1. 1; every positive power of 1 is 1.
2. 2; the principal square root of 4.
3. 2; $2^3 = 8$.
4. 4; cube root first, then square.
5. 27; $3^3 = 27$.
6. 32; $2^5 = 32$.
7. $3/4$; the expression is $2^{3/4}$.
8. $1/2$, $2/4$, $3/6$, $4/8$, $5/10$.
9. $3^{2/3}$; $2/3 > 1/2$ and $3 > 1$.
10. Yes; its exponent lies between 0 and 1.
11. -2; the real cube root.
12. 4; $(-2)^2 = 4$.
13. -2; reduce $2/6$ to $1/3$.
14. 4; reduce $4/6$ to $2/3$.
15. No; $3/6$ reduces to $1/2$.
16. $3i$; the principal square root.
17. $-8i$; magnitude 8, angle $3\pi/2$.
18. 1; odd root followed by an even power.
19. $\cos(4\pi/5) + i \sin(4\pi/5)$.
20. 0; the exponent is positive.
21. 16; add the exponents to obtain 2.
22. 9; subtract the exponents to obtain 1.
23. 2 is not a perfect square integer.
24. $\sqrt{10}$ m; approximately 3.162278 m.
25. 2 m; the cube root of 8 m^3 .
26. 4; $8^{2/3} = 4$.
27. $2^{1/3} \approx 1.259921$ per period.
28. 16 and -16; parentheses change the base.
29. 100 forms and 63 distinct rational exponents.
30. $10 \times 10^{1/9} \approx 12.915497$.

EXTENSION: A COMPLETE SAMPLE

For base -8, exponent 1 gives -8 in both conventions. Exponent $1/3$ gives real value -2 and principal complex value $1 + (\sqrt{3})i$. Both values cube to -8; they select different cube roots. A real value exists exactly when the reduced denominator is odd.

Glossary, index & references

TERM	DEFINITION / WHERE TO LOOK
Approximation	A nearby numerical value, marked $\approx$; pages 9, 13-64.
Argument	An angle locating a nonzero complex number; page 7.
Branch	A consistent selection among possible complex values; page 7.
Complex number	A number $x + yi$ with real x and y ; page 7.
Domain	The allowed inputs for a stated convention; pages 6-8.
Equivalent fractions	Different forms of one rational number; page 4.
Exact value	A value stated without rounding; pages 3, 5, 9.
Imaginary unit	The number i , with $i^2 = -1$; page 7.
Lowest terms	A fraction with coprime numerator and denominator; pages 4, 6.
Principal value	A value selected by an explicit root or angle convention; page 7.
Rational number	A quotient of integers with nonzero denominator; page 4.
Real odd-root value	The real root choice for an odd reduced denominator; page 6.

References and further reading

[1] OpenStax, *College Algebra 2e*, §1.3, “Radicals and Rational Exponents.” Positive-base rules and radical notation. openstax.org/books/college-algebra-2e/pages/1-3-radicals-and-rational-exponents

[2] Lane Community College, ORCCA, §13.2, “Radical Expressions and Rational Exponents.” Real odd roots and calculator conventions.

math.oer.lanecollege.edu/orcca/section-radical-expressions-and-rational-exponents.html

[3] NIST, *Digital Library of Mathematical Functions*, §4.2, “Definitions,” especially logarithms and powers. This book uses the upper-side choice $\text{Arg}(-a) = \pi$.

dlmf.nist.gov/4.2 • References consulted September 30, 2026.

HOW TO USE THE ATLAS

At each magnitude, read the three negative-base pages followed by the two positive-base pages. The reduced exponent is included in every entry. The original fraction stays visible so that all 100 mathematical combinations can be compared. Each positive-base result is also its principal complex value, with imaginary part zero.

Numerical method: positive magnitudes were computed with 60-digit decimal arithmetic before rounding. Exact integer roots were checked separately. Complex components use the stated polar formula; integer and half-integer angles receive exact axis treatment.

Base 0: complete exponent table

Every listed exponent is positive. Therefore every value below is exactly zero. The cases 0 and negative exponents are outside this atlas.

TERM	r/s	VALUE	TERM	r/s	VALUE
$0^{1/2}$	1/2	= 0	$0^{3/6}$	1/2	= 0
$0^{1/3}$	1/3	= 0	$0^{3/7}$	3/7	= 0
$0^{1/4}$	1/4	= 0	$0^{3/8}$	3/8	= 0
$0^{1/5}$	1/5	= 0	$0^{3/9}$	1/3	= 0
$0^{1/6}$	1/6	= 0	$0^{3/10}$	3/10	= 0
$0^{1/7}$	1/7	= 0	$0^{4/1}$	4	= 0
$0^{1/8}$	1/8	= 0	$0^{4/2}$	2	= 0
$0^{1/9}$	1/9	= 0	$0^{4/3}$	4/3	= 0
$0^{1/10}$	1/10	= 0	$0^{4/4}$	1	= 0
$0^{1/1}$	1	= 0	$0^{4/5}$	4/5	= 0
$0^{2/1}$	2	= 0	$0^{4/6}$	2/3	= 0
$0^{2/2}$	1	= 0	$0^{4/7}$	4/7	= 0
$0^{2/3}$	2/3	= 0	$0^{4/8}$	1/2	= 0
$0^{2/4}$	1/2	= 0	$0^{4/9}$	4/9	= 0
$0^{2/5}$	2/5	= 0	$0^{4/10}$	2/5	= 0
$0^{2/6}$	1/3	= 0	$0^{5/1}$	5	= 0
$0^{2/7}$	2/7	= 0	$0^{5/2}$	5/2	= 0
$0^{2/8}$	1/4	= 0	$0^{5/3}$	5/3	= 0
$0^{2/9}$	2/9	= 0	$0^{5/4}$	5/4	= 0
$0^{2/10}$	1/5	= 0	$0^{5/5}$	1	= 0
$0^{3/1}$	3	= 0	$0^{5/6}$	5/6	= 0
$0^{3/2}$	3/2	= 0	$0^{5/7}$	5/7	= 0
$0^{3/3}$	1	= 0	$0^{5/8}$	5/8	= 0
$0^{3/4}$	3/4	= 0	$0^{5/9}$	5/9	= 0
$0^{3/5}$	3/5	= 0	$0^{5/10}$	1/2	= 0

ZERO CHECK

For any p/q in this table, $p > 0$ and $q > 0$. The q th root of zero is zero, and a positive integer power of zero stays zero.

Base 0: complete exponent table

Every listed exponent is positive. Therefore every value below is exactly zero. The cases 0 and negative exponents are outside this atlas.

TERM	r/s	VALUE	TERM	r/s	VALUE
$0^{6/1}$	6	= 0	$0^{8/6}$	4/3	= 0
$0^{6/2}$	3	= 0	$0^{8/7}$	8/7	= 0
$0^{6/3}$	2	= 0	$0^{8/8}$	1	= 0
$0^{6/4}$	3/2	= 0	$0^{8/9}$	8/9	= 0
$0^{6/5}$	6/5	= 0	$0^{8/10}$	4/5	= 0
$0^{6/6}$	1	= 0	$0^{9/1}$	9	= 0
$0^{6/7}$	6/7	= 0	$0^{9/2}$	9/2	= 0
$0^{6/8}$	3/4	= 0	$0^{9/3}$	3	= 0
$0^{6/9}$	2/3	= 0	$0^{9/4}$	9/4	= 0
$0^{6/10}$	3/5	= 0	$0^{9/5}$	9/5	= 0
$0^{7/1}$	7	= 0	$0^{9/6}$	3/2	= 0
$0^{7/2}$	7/2	= 0	$0^{9/7}$	9/7	= 0
$0^{7/3}$	7/3	= 0	$0^{9/8}$	9/8	= 0
$0^{7/4}$	7/4	= 0	$0^{9/9}$	1	= 0
$0^{7/5}$	7/5	= 0	$0^{9/10}$	9/10	= 0
$0^{7/6}$	7/6	= 0	$0^{10/1}$	10	= 0
$0^{7/7}$	1	= 0	$0^{10/2}$	5	= 0
$0^{7/8}$	7/8	= 0	$0^{10/3}$	10/3	= 0
$0^{7/9}$	7/9	= 0	$0^{10/4}$	5/2	= 0
$0^{7/10}$	7/10	= 0	$0^{10/5}$	2	= 0
$0^{8/1}$	8	= 0	$0^{10/6}$	5/3	= 0
$0^{8/2}$	4	= 0	$0^{10/7}$	10/7	= 0
$0^{8/3}$	8/3	= 0	$0^{10/8}$	5/4	= 0
$0^{8/4}$	2	= 0	$0^{10/10}$	1	= 0
$0^{8/5}$	8/5	= 0	$0^{10/9}$	10/9	= 0

ZERO CHECK

Zero is the unique complex solution of $z^s = 0$. The logarithm of zero is undefined, so its powers here are defined directly by roots and positive powers, not by the logarithm formula.

Base -1: real & principal complex

Reduce p/q to r/s. Odd s permits a real value; even s does not. The principal complex column uses $\text{Arg}(-1) = \pi$.

TERM	r/s	REAL VALUE	PRINCIPAL COMPLEX VALUE
$(-1)^{1/2}$	1/2	No real value	= i
$(-1)^{1/3}$	1/3	= -1	$\approx 0.500000 + 0.866025i$
$(-1)^{1/4}$	1/4	No real value	$\approx 0.707107 + 0.707107i$
$(-1)^{1/5}$	1/5	= -1	$\approx 0.809017 + 0.587785i$
$(-1)^{1/6}$	1/6	No real value	$\approx 0.866025 + 0.500000i$
$(-1)^{1/7}$	1/7	= -1	$\approx 0.900969 + 0.433884i$
$(-1)^{1/8}$	1/8	No real value	$\approx 0.923880 + 0.382683i$
$(-1)^{1/9}$	1/9	= -1	$\approx 0.939693 + 0.342020i$
$(-1)^{1/10}$	1/10	No real value	$\approx 0.951057 + 0.309017i$
$(-1)^{1/1}$	1	= -1	= -1
$(-1)^{2/1}$	2	= 1	= 1
$(-1)^{2/2}$	1	= -1	= -1
$(-1)^{2/3}$	2/3	= 1	$\approx -0.500000 + 0.866025i$
$(-1)^{2/4}$	1/2	No real value	= i
$(-1)^{2/5}$	2/5	= 1	$\approx 0.309017 + 0.951057i$
$(-1)^{2/6}$	1/3	= -1	$\approx 0.500000 + 0.866025i$
$(-1)^{2/7}$	2/7	= 1	$\approx 0.623490 + 0.781831i$
$(-1)^{2/8}$	1/4	No real value	$\approx 0.707107 + 0.707107i$
$(-1)^{2/9}$	2/9	= 1	$\approx 0.766044 + 0.642788i$
$(-1)^{2/10}$	1/5	= -1	$\approx 0.809017 + 0.587785i$
$(-1)^{3/1}$	3	= -1	= -1
$(-1)^{3/2}$	3/2	No real value	= -i
$(-1)^{3/3}$	1	= -1	= -1
$(-1)^{3/4}$	3/4	No real value	$\approx -0.707107 + 0.707107i$
$(-1)^{3/5}$	3/5	= -1	$\approx -0.309017 + 0.951057i$
$(-1)^{3/6}$	1/2	No real value	= i
$(-1)^{3/7}$	3/7	= -1	$\approx 0.222521 + 0.974928i$
$(-1)^{3/8}$	3/8	No real value	$\approx 0.382683 + 0.923880i$
$(-1)^{3/9}$	1/3	= -1	$\approx 0.500000 + 0.866025i$
$(-1)^{3/10}$	3/10	No real value	$\approx 0.587785 + 0.809017i$
$(-1)^{4/1}$	4	= 1	= 1
$(-1)^{4/2}$	2	= 1	= 1
$(-1)^{4/3}$	4/3	= 1	$\approx -0.500000 - 0.866025i$
$(-1)^{4/4}$	1	= -1	= -1

Exact definition: $P = 1^{r/s} [\cos(\pi r/s) + i \sin(\pi r/s)]$. A nonreal principal value can coexist with a real odd-root value.

Base -1: real & principal complex

Reduce p/q to r/s. Odd s permits a real value; even s does not. The principal complex column uses $\text{Arg}(-1) = \pi$.

TERM	r/s	REAL VALUE	PRINCIPAL COMPLEX VALUE
$(-1)^{4/5}$	4/5	= 1	$\approx -0.809017 + 0.587785i$
$(-1)^{4/6}$	2/3	= 1	$\approx -0.500000 + 0.866025i$
$(-1)^{4/7}$	4/7	= 1	$\approx -0.222521 + 0.974928i$
$(-1)^{4/8}$	1/2	No real value	= i
$(-1)^{4/9}$	4/9	= 1	$\approx 0.173648 + 0.984808i$
$(-1)^{4/10}$	2/5	= 1	$\approx 0.309017 + 0.951057i$
$(-1)^{5/1}$	5	= -1	= -1
$(-1)^{5/2}$	5/2	No real value	= i
$(-1)^{5/3}$	5/3	= -1	$\approx 0.500000 - 0.866025i$
$(-1)^{5/4}$	5/4	No real value	$\approx -0.707107 - 0.707107i$
$(-1)^{5/5}$	1	= -1	= -1
$(-1)^{5/6}$	5/6	No real value	$\approx -0.866025 + 0.500000i$
$(-1)^{5/7}$	5/7	= -1	$\approx -0.623490 + 0.781831i$
$(-1)^{5/8}$	5/8	No real value	$\approx -0.382683 + 0.923880i$
$(-1)^{5/9}$	5/9	= -1	$\approx -0.173648 + 0.984808i$
$(-1)^{5/10}$	1/2	No real value	= i
$(-1)^{6/1}$	6	= 1	= 1
$(-1)^{6/2}$	3	= -1	= -1
$(-1)^{6/3}$	2	= 1	= 1
$(-1)^{6/4}$	3/2	No real value	= -i
$(-1)^{6/5}$	6/5	= 1	$\approx -0.809017 - 0.587785i$
$(-1)^{6/6}$	1	= -1	= -1
$(-1)^{6/7}$	6/7	= 1	$\approx -0.900969 + 0.433884i$
$(-1)^{6/8}$	3/4	No real value	$\approx -0.707107 + 0.707107i$
$(-1)^{6/9}$	2/3	= 1	$\approx -0.500000 + 0.866025i$
$(-1)^{6/10}$	3/5	= -1	$\approx -0.309017 + 0.951057i$
$(-1)^{7/1}$	7	= -1	= -1
$(-1)^{7/2}$	7/2	No real value	= -i
$(-1)^{7/3}$	7/3	= -1	$\approx 0.500000 + 0.866025i$
$(-1)^{7/4}$	7/4	No real value	$\approx 0.707107 - 0.707107i$
$(-1)^{7/5}$	7/5	= -1	$\approx -0.309017 - 0.951057i$
$(-1)^{7/6}$	7/6	No real value	$\approx -0.866025 - 0.500000i$
$(-1)^{7/7}$	1	= -1	= -1

The real column uses $(-1)^r \times 1^{r/s}$ when s is odd. Even reduced numerators give positive real values; odd ones give negative real values.

Base -1: real & principal complex

Reduce p/q to r/s. Odd s permits a real value; even s does not. The principal complex column uses $\text{Arg}(-1) = \pi$.

TERM	r/s	REAL VALUE	PRINCIPAL COMPLEX VALUE
$(-1)^{7/8}$	7/8	No real value	$\approx -0.923880 + 0.382683i$
$(-1)^{7/9}$	7/9	= -1	$\approx -0.766044 + 0.642788i$
$(-1)^{7/10}$	7/10	No real value	$\approx -0.587785 + 0.809017i$
$(-1)^{8/1}$	8	= 1	= 1
$(-1)^{8/2}$	4	= 1	= 1
$(-1)^{8/3}$	8/3	= 1	$\approx -0.500000 + 0.866025i$
$(-1)^{8/4}$	2	= 1	= 1
$(-1)^{8/5}$	8/5	= 1	$\approx 0.309017 - 0.951057i$
$(-1)^{8/6}$	4/3	= 1	$\approx -0.500000 - 0.866025i$
$(-1)^{8/7}$	8/7	= 1	$\approx -0.900969 - 0.433884i$
$(-1)^{8/8}$	1	= -1	= -1
$(-1)^{8/9}$	8/9	= 1	$\approx -0.939693 + 0.342020i$
$(-1)^{8/10}$	4/5	= 1	$\approx -0.809017 + 0.587785i$
$(-1)^{9/1}$	9	= -1	= -1
$(-1)^{9/2}$	9/2	No real value	= i
$(-1)^{9/3}$	3	= -1	= -1
$(-1)^{9/4}$	9/4	No real value	$\approx 0.707107 + 0.707107i$
$(-1)^{9/5}$	9/5	= -1	$\approx 0.809017 - 0.587785i$
$(-1)^{9/6}$	3/2	No real value	= -i
$(-1)^{9/7}$	9/7	= -1	$\approx -0.623490 - 0.781831i$
$(-1)^{9/8}$	9/8	No real value	$\approx -0.923880 - 0.382683i$
$(-1)^{9/9}$	1	= -1	= -1
$(-1)^{9/10}$	9/10	No real value	$\approx -0.951057 + 0.309017i$
$(-1)^{10/1}$	10	= 1	= 1
$(-1)^{10/2}$	5	= -1	= -1
$(-1)^{10/3}$	10/3	= 1	$\approx -0.500000 - 0.866025i$
$(-1)^{10/4}$	5/2	No real value	= i
$(-1)^{10/5}$	2	= 1	= 1
$(-1)^{10/6}$	5/3	= -1	$\approx 0.500000 - 0.866025i$
$(-1)^{10/7}$	10/7	= 1	$\approx -0.222521 - 0.974928i$
$(-1)^{10/8}$	5/4	No real value	$\approx -0.707107 - 0.707107i$
$(-1)^{10/10}$	1	= -1	= -1
$(-1)^{10/9}$	10/9	= 1	$\approx -0.939693 - 0.342020i$

Equivalent fractions give the same value within each convention. Principal complex values agree with real values at integer exponents; noninteger powers with odd reduced denominators select different branches.

Base 1: complete exponent table

Positive real convention. All decimal values marked $\approx$ are rounded to six places. Read down the left block, then down the right block.

TERM	r/s	VALUE	TERM	r/s	VALUE
$1^{1/2}$	1/2	= 1	$1^{3/6}$	1/2	= 1
$1^{1/3}$	1/3	= 1	$1^{3/7}$	3/7	= 1
$1^{1/4}$	1/4	= 1	$1^{3/8}$	3/8	= 1
$1^{1/5}$	1/5	= 1	$1^{3/9}$	1/3	= 1
$1^{1/6}$	1/6	= 1	$1^{3/10}$	3/10	= 1
$1^{1/7}$	1/7	= 1	$1^{4/1}$	4	= 1
$1^{1/8}$	1/8	= 1	$1^{4/2}$	2	= 1
$1^{1/9}$	1/9	= 1	$1^{4/3}$	4/3	= 1
$1^{1/10}$	1/10	= 1	$1^{4/4}$	1	= 1
$1^{1/1}$	1	= 1	$1^{4/5}$	4/5	= 1
$1^{2/1}$	2	= 1	$1^{4/6}$	2/3	= 1
$1^{2/2}$	1	= 1	$1^{4/7}$	4/7	= 1
$1^{2/3}$	2/3	= 1	$1^{4/8}$	1/2	= 1
$1^{2/4}$	1/2	= 1	$1^{4/9}$	4/9	= 1
$1^{2/5}$	2/5	= 1	$1^{4/10}$	2/5	= 1
$1^{2/6}$	1/3	= 1	$1^{5/1}$	5	= 1
$1^{2/7}$	2/7	= 1	$1^{5/2}$	5/2	= 1
$1^{2/8}$	1/4	= 1	$1^{5/3}$	5/3	= 1
$1^{2/9}$	2/9	= 1	$1^{5/4}$	5/4	= 1
$1^{2/10}$	1/5	= 1	$1^{5/5}$	1	= 1
$1^{3/1}$	3	= 1	$1^{5/6}$	5/6	= 1
$1^{3/2}$	3/2	= 1	$1^{5/7}$	5/7	= 1
$1^{3/3}$	1	= 1	$1^{5/8}$	5/8	= 1
$1^{3/4}$	3/4	= 1	$1^{5/9}$	5/9	= 1
$1^{3/5}$	3/5	= 1	$1^{5/10}$	1/2	= 1

PATTERN TO NOTICE

Every value is exactly 1. Changing the positive rational exponent cannot change the value of the base one.

Base 1: complete exponent table

Positive real convention. All decimal values marked $\approx$ are rounded to six places. Read down the left block, then down the right block.

TERM	r/s	VALUE	TERM	r/s	VALUE
$1^{6/1}$	6	= 1	$1^{8/6}$	$4/3$	= 1
$1^{6/2}$	3	= 1	$1^{8/7}$	$8/7$	= 1
$1^{6/3}$	2	= 1	$1^{8/8}$	1	= 1
$1^{6/4}$	$3/2$	= 1	$1^{8/9}$	$8/9$	= 1
$1^{6/5}$	$6/5$	= 1	$1^{8/10}$	$4/5$	= 1
$1^{6/6}$	1	= 1	$1^{9/1}$	9	= 1
$1^{6/7}$	$6/7$	= 1	$1^{9/2}$	$9/2$	= 1
$1^{6/8}$	$3/4$	= 1	$1^{9/3}$	3	= 1
$1^{6/9}$	$2/3$	= 1	$1^{9/4}$	$9/4$	= 1
$1^{6/10}$	$3/5$	= 1	$1^{9/5}$	$9/5$	= 1
$1^{7/1}$	7	= 1	$1^{9/6}$	$3/2$	= 1
$1^{7/2}$	$7/2$	= 1	$1^{9/7}$	$9/7$	= 1
$1^{7/3}$	$7/3$	= 1	$1^{9/8}$	$9/8$	= 1
$1^{7/4}$	$7/4$	= 1	$1^{9/9}$	1	= 1
$1^{7/5}$	$7/5$	= 1	$1^{9/10}$	$9/10$	= 1
$1^{7/6}$	$7/6$	= 1	$1^{10/1}$	10	= 1
$1^{7/7}$	1	= 1	$1^{10/2}$	5	= 1
$1^{7/8}$	$7/8$	= 1	$1^{10/3}$	$10/3$	= 1
$1^{7/9}$	$7/9$	= 1	$1^{10/4}$	$5/2$	= 1
$1^{7/10}$	$7/10$	= 1	$1^{10/5}$	2	= 1
$1^{8/1}$	8	= 1	$1^{10/6}$	$5/3$	= 1
$1^{8/2}$	4	= 1	$1^{10/7}$	$10/7$	= 1
$1^{8/3}$	$8/3$	= 1	$1^{10/8}$	$5/4$	= 1
$1^{8/4}$	2	= 1	$1^{10/10}$	1	= 1
$1^{8/5}$	$8/5$	= 1	$1^{10/9}$	$10/9$	= 1

PATTERN TO NOTICE

Every value is exactly 1. Changing the positive rational exponent cannot change the value of the base one.

Base -2: real & principal complex

Reduce p/q to r/s. Odd s permits a real value; even s does not. The principal complex column uses $\text{Arg}(-2) = \pi$.

TERM	r/s	REAL VALUE	PRINCIPAL COMPLEX VALUE
$(-2)^{1/2}$	1/2	No real value	$\approx 1.414214i$
$(-2)^{1/3}$	1/3	≈ -1.259921	$\approx 0.629961 + 1.091124i$
$(-2)^{1/4}$	1/4	No real value	$\approx 0.840896 + 0.840896i$
$(-2)^{1/5}$	1/5	≈ -1.148698	$\approx 0.929316 + 0.675188i$
$(-2)^{1/6}$	1/6	No real value	$\approx 0.972081 + 0.561231i$
$(-2)^{1/7}$	1/7	≈ -1.104090	$\approx 0.994750 + 0.479046i$
$(-2)^{1/8}$	1/8	No real value	$\approx 1.007498 + 0.417319i$
$(-2)^{1/9}$	1/9	≈ -1.080060	$\approx 1.014924 + 0.369402i$
$(-2)^{1/10}$	1/10	No real value	$\approx 1.019317 + 0.331196i$
$(-2)^{1/1}$	1	$= -2$	$= -2$
$(-2)^{2/1}$	2	$= 4$	$= 4$
$(-2)^{2/2}$	1	$= -2$	$= -2$
$(-2)^{2/3}$	2/3	≈ 1.587401	$\approx -0.793701 + 1.374730i$
$(-2)^{2/4}$	1/2	No real value	$\approx 1.414214i$
$(-2)^{2/5}$	2/5	≈ 1.319508	$\approx 0.407750 + 1.254927i$
$(-2)^{2/6}$	1/3	≈ -1.259921	$\approx 0.629961 + 1.091124i$
$(-2)^{2/7}$	2/7	≈ 1.219014	$\approx 0.760043 + 0.953063i$
$(-2)^{2/8}$	1/4	No real value	$\approx 0.840896 + 0.840896i$
$(-2)^{2/9}$	2/9	≈ 1.166529	$\approx 0.893613 + 0.749830i$
$(-2)^{2/10}$	1/5	≈ -1.148698	$\approx 0.929316 + 0.675188i$
$(-2)^{3/1}$	3	$= -8$	$= -8$
$(-2)^{3/2}$	3/2	No real value	$\approx -2.828427i$
$(-2)^{3/3}$	1	$= -2$	$= -2$
$(-2)^{3/4}$	3/4	No real value	$\approx -1.189207 + 1.189207i$
$(-2)^{3/5}$	3/5	≈ -1.515717	$\approx -0.468382 + 1.441532i$
$(-2)^{3/6}$	1/2	No real value	$\approx 1.414214i$
$(-2)^{3/7}$	3/7	≈ -1.345900	$\approx 0.299491 + 1.312156i$
$(-2)^{3/8}$	3/8	No real value	$\approx 0.496279 + 1.198124i$
$(-2)^{3/9}$	1/3	≈ -1.259921	$\approx 0.629961 + 1.091124i$
$(-2)^{3/10}$	3/10	No real value	$\approx 0.723649 + 0.996017i$
$(-2)^{4/1}$	4	$= 16$	$= 16$
$(-2)^{4/2}$	2	$= 4$	$= 4$
$(-2)^{4/3}$	4/3	≈ 2.519842	$\approx -1.259921 - 2.182247i$
$(-2)^{4/4}$	1	$= -2$	$= -2$

Exact definition: $P = 2^{r/s} [\cos(\pi r/s) + i \sin(\pi r/s)]$. A nonreal principal value can coexist with a real odd-root value.

Base -2: real & principal complex

Reduce p/q to r/s. Odd s permits a real value; even s does not. The principal complex column uses $\text{Arg}(-2) = \pi$.

TERM	r/s	REAL VALUE	PRINCIPAL COMPLEX VALUE
$(-2)^{4/5}$	4/5	≈ 1.741101	$\approx -1.408580 + 1.023394i$
$(-2)^{4/6}$	2/3	≈ 1.587401	$\approx -0.793701 + 1.374730i$
$(-2)^{4/7}$	4/7	≈ 1.485994	$\approx -0.330665 + 1.448737i$
$(-2)^{4/8}$	1/2	No real value	$\approx 1.414214i$
$(-2)^{4/9}$	4/9	≈ 1.360790	$\approx 0.236299 + 1.340117i$
$(-2)^{4/10}$	2/5	≈ 1.319508	$\approx 0.407750 + 1.254927i$
$(-2)^{5/1}$	5	$= -32$	$= -32$
$(-2)^{5/2}$	5/2	No real value	$\approx 5.656854i$
$(-2)^{5/3}$	5/3	≈ -3.174802	$\approx 1.587401 - 2.749459i$
$(-2)^{5/4}$	5/4	No real value	$\approx -1.681793 - 1.681793i$
$(-2)^{5/5}$	1	$= -2$	$= -2$
$(-2)^{5/6}$	5/6	No real value	$\approx -1.543082 + 0.890899i$
$(-2)^{5/7}$	5/7	≈ -1.640671	$\approx -1.022941 + 1.282728i$
$(-2)^{5/8}$	5/8	No real value	$\approx -0.590179 + 1.424817i$
$(-2)^{5/9}$	5/9	≈ -1.469734	$\approx -0.255217 + 1.447406i$
$(-2)^{5/10}$	1/2	No real value	$\approx 1.414214i$
$(-2)^{6/1}$	6	$= 64$	$= 64$
$(-2)^{6/2}$	3	$= -8$	$= -8$
$(-2)^{6/3}$	2	$= 4$	$= 4$
$(-2)^{6/4}$	3/2	No real value	$\approx -2.828427i$
$(-2)^{6/5}$	6/5	≈ 2.297397	$\approx -1.858633 - 1.350376i$
$(-2)^{6/6}$	1	$= -2$	$= -2$
$(-2)^{6/7}$	6/7	≈ 1.811447	$\approx -1.632058 + 0.785958i$
$(-2)^{6/8}$	3/4	No real value	$\approx -1.189207 + 1.189207i$
$(-2)^{6/9}$	2/3	≈ 1.587401	$\approx -0.793701 + 1.374730i$
$(-2)^{6/10}$	3/5	≈ -1.515717	$\approx -0.468382 + 1.441532i$
$(-2)^{7/1}$	7	$= -128$	$= -128$
$(-2)^{7/2}$	7/2	No real value	$\approx -11.313708i$
$(-2)^{7/3}$	7/3	≈ -5.039684	$\approx 2.519842 + 4.364495i$
$(-2)^{7/4}$	7/4	No real value	$\approx 2.378414 - 2.378414i$
$(-2)^{7/5}$	7/5	≈ -2.639016	$\approx -0.815501 - 2.509853i$
$(-2)^{7/6}$	7/6	No real value	$\approx -1.944161 - 1.122462i$
$(-2)^{7/7}$	1	$= -2$	$= -2$

The real column uses $(-1)^r \times 2^{r/s}$ when s is odd. Even reduced numerators give positive real values; odd ones give negative real values.

Base -2: real & principal complex

Reduce p/q to r/s. Odd s permits a real value; even s does not. The principal complex column uses $\text{Arg}(-2) = \pi$.

TERM	r/s	REAL VALUE	PRINCIPAL COMPLEX VALUE
$(-2)^{7/8}$	7/8	No real value	$\approx -1.694403 + 0.701845i$
$(-2)^{7/9}$	7/9	≈ -1.714488	$\approx -1.313374 + 1.102052i$
$(-2)^{7/10}$	7/10	No real value	$\approx -0.954860 + 1.314252i$
$(-2)^{8/1}$	8	$= 256$	$= 256$
$(-2)^{8/2}$	4	$= 16$	$= 16$
$(-2)^{8/3}$	8/3	≈ 6.349604	$\approx -3.174802 + 5.498919i$
$(-2)^{8/4}$	2	$= 4$	$= 4$
$(-2)^{8/5}$	8/5	≈ 3.031433	$\approx 0.936764 - 2.883064i$
$(-2)^{8/6}$	4/3	≈ 2.519842	$\approx -1.259921 - 2.182247i$
$(-2)^{8/7}$	8/7	≈ 2.208179	$\approx -1.989501 - 0.958093i$
$(-2)^{8/8}$	1	$= -2$	$= -2$
$(-2)^{8/9}$	8/9	≈ 1.851749	$\approx -1.740075 + 0.633336i$
$(-2)^{8/10}$	4/5	≈ 1.741101	$\approx -1.408580 + 1.023394i$
$(-2)^{9/1}$	9	$= -512$	$= -512$
$(-2)^{9/2}$	9/2	No real value	$\approx 22.627417i$
$(-2)^{9/3}$	3	$= -8$	$= -8$
$(-2)^{9/4}$	9/4	No real value	$\approx 3.363586 + 3.363586i$
$(-2)^{9/5}$	9/5	≈ -3.482202	$\approx 2.817161 - 2.046787i$
$(-2)^{9/6}$	3/2	No real value	$\approx -2.828427i$
$(-2)^{9/7}$	9/7	≈ -2.438027	$\approx -1.520085 - 1.906127i$
$(-2)^{9/8}$	9/8	No real value	$\approx -2.014996 - 0.834638i$
$(-2)^{9/9}$	1	$= -2$	$= -2$
$(-2)^{9/10}$	9/10	No real value	$\approx -1.774734 + 0.576646i$
$(-2)^{10/1}$	10	$= 1,024$	$= 1,024$
$(-2)^{10/2}$	5	$= -32$	$= -32$
$(-2)^{10/3}$	10/3	≈ 10.079368	$\approx -5.039684 - 8.728989i$
$(-2)^{10/4}$	5/2	No real value	$\approx 5.656854i$
$(-2)^{10/5}$	2	$= 4$	$= 4$
$(-2)^{10/6}$	5/3	≈ -3.174802	$\approx 1.587401 - 2.749459i$
$(-2)^{10/7}$	10/7	≈ 2.691800	$\approx -0.598982 - 2.624311i$
$(-2)^{10/8}$	5/4	No real value	$\approx -1.681793 - 1.681793i$
$(-2)^{10/10}$	1	$= -2$	$= -2$
$(-2)^{10/9}$	10/9	≈ 2.160119	$\approx -2.029848 - 0.738804i$

Equivalent fractions give the same value within each convention. Principal complex values agree with real values at integer exponents; noninteger powers with odd reduced denominators select different branches.

Base 2: complete exponent table

Positive real convention. All decimal values marked $\approx$ are rounded to six places. Read down the left block, then down the right block.

TERM	r/s	VALUE	TERM	r/s	VALUE
$2^{1/2}$	1/2	≈ 1.414214	$2^{3/6}$	1/2	≈ 1.414214
$2^{1/3}$	1/3	≈ 1.259921	$2^{3/7}$	3/7	≈ 1.345900
$2^{1/4}$	1/4	≈ 1.189207	$2^{3/8}$	3/8	≈ 1.296840
$2^{1/5}$	1/5	≈ 1.148698	$2^{3/9}$	1/3	≈ 1.259921
$2^{1/6}$	1/6	≈ 1.122462	$2^{3/10}$	3/10	≈ 1.231144
$2^{1/7}$	1/7	≈ 1.104090	$2^{4/1}$	4	$= 16$
$2^{1/8}$	1/8	≈ 1.090508	$2^{4/2}$	2	$= 4$
$2^{1/9}$	1/9	≈ 1.080060	$2^{4/3}$	4/3	≈ 2.519842
$2^{1/10}$	1/10	≈ 1.071773	$2^{4/4}$	1	$= 2$
$2^{1/1}$	1	$= 2$	$2^{4/5}$	4/5	≈ 1.741101
$2^{2/1}$	2	$= 4$	$2^{4/6}$	2/3	≈ 1.587401
$2^{2/2}$	1	$= 2$	$2^{4/7}$	4/7	≈ 1.485994
$2^{2/3}$	2/3	≈ 1.587401	$2^{4/8}$	1/2	≈ 1.414214
$2^{2/4}$	1/2	≈ 1.414214	$2^{4/9}$	4/9	≈ 1.360790
$2^{2/5}$	2/5	≈ 1.319508	$2^{4/10}$	2/5	≈ 1.319508
$2^{2/6}$	1/3	≈ 1.259921	$2^{5/1}$	5	$= 32$
$2^{2/7}$	2/7	≈ 1.219014	$2^{5/2}$	5/2	≈ 5.656854
$2^{2/8}$	1/4	≈ 1.189207	$2^{5/3}$	5/3	≈ 3.174802
$2^{2/9}$	2/9	≈ 1.166529	$2^{5/4}$	5/4	≈ 2.378414
$2^{2/10}$	1/5	≈ 1.148698	$2^{5/5}$	1	$= 2$
$2^{3/1}$	3	$= 8$	$2^{5/6}$	5/6	≈ 1.781797
$2^{3/2}$	3/2	≈ 2.828427	$2^{5/7}$	5/7	≈ 1.640671
$2^{3/3}$	1	$= 2$	$2^{5/8}$	5/8	≈ 1.542211
$2^{3/4}$	3/4	≈ 1.681793	$2^{5/9}$	5/9	≈ 1.469734
$2^{3/5}$	3/5	≈ 1.515717	$2^{5/10}$	1/2	≈ 1.414214

PATTERN TO NOTICE

The base 2 is prime. A reduced denominator greater than 1 produces an irrational real result; denominator 1 produces an integer.

Base 2: complete exponent table

Positive real convention. All decimal values marked $\approx$ are rounded to six places. Read down the left block, then down the right block.

TERM	r/s	VALUE
$2^{6/1}$	6	= 64
$2^{6/2}$	3	= 8
$2^{6/3}$	2	= 4
$2^{6/4}$	3/2	≈ 2.828427
$2^{6/5}$	6/5	≈ 2.297397
$2^{6/6}$	1	= 2
$2^{6/7}$	6/7	≈ 1.811447
$2^{6/8}$	3/4	≈ 1.681793
$2^{6/9}$	2/3	≈ 1.587401
$2^{6/10}$	3/5	≈ 1.515717
$2^{7/1}$	7	= 128
$2^{7/2}$	7/2	≈ 11.313708
$2^{7/3}$	7/3	≈ 5.039684
$2^{7/4}$	7/4	≈ 3.363586
$2^{7/5}$	7/5	≈ 2.639016
$2^{7/6}$	7/6	≈ 2.244924
$2^{7/7}$	1	= 2
$2^{7/8}$	7/8	≈ 1.834008
$2^{7/9}$	7/9	≈ 1.714488
$2^{7/10}$	7/10	≈ 1.624505
$2^{8/1}$	8	= 256
$2^{8/2}$	4	= 16
$2^{8/3}$	8/3	≈ 6.349604
$2^{8/4}$	2	= 4
$2^{8/5}$	8/5	≈ 3.031433

TERM	r/s	VALUE
$2^{8/6}$	4/3	≈ 2.519842
$2^{8/7}$	8/7	≈ 2.208179
$2^{8/8}$	1	= 2
$2^{8/9}$	8/9	≈ 1.851749
$2^{8/10}$	4/5	≈ 1.741101
$2^{9/1}$	9	= 512
$2^{9/2}$	9/2	≈ 22.627417
$2^{9/3}$	3	= 8
$2^{9/4}$	9/4	≈ 4.756828
$2^{9/5}$	9/5	≈ 3.482202
$2^{9/6}$	3/2	≈ 2.828427
$2^{9/7}$	9/7	≈ 2.438027
$2^{9/8}$	9/8	≈ 2.181015
$2^{9/9}$	1	= 2
$2^{9/10}$	9/10	≈ 1.866066
$2^{10/1}$	10	= 1,024
$2^{10/2}$	5	= 32
$2^{10/3}$	10/3	≈ 10.079368
$2^{10/4}$	5/2	≈ 5.656854
$2^{10/5}$	2	= 4
$2^{10/6}$	5/3	≈ 3.174802
$2^{10/7}$	10/7	≈ 2.691800
$2^{10/8}$	5/4	≈ 2.378414
$2^{10/10}$	1	= 2
$2^{10/9}$	10/9	≈ 2.160119

PATTERN TO NOTICE

The base 2 is prime. A reduced denominator greater than 1 produces an irrational real result; denominator 1 produces an integer.

Base -3: real & principal complex

Reduce p/q to r/s. Odd s permits a real value; even s does not. The principal complex column uses $\text{Arg}(-3) = \pi$.

TERM	r/s	REAL VALUE	PRINCIPAL COMPLEX VALUE
$(-3)^{1/2}$	1/2	No real value	$\approx 1.732051i$
$(-3)^{1/3}$	1/3	≈ -1.442250	$\approx 0.721125 + 1.249025i$
$(-3)^{1/4}$	1/4	No real value	$\approx 0.930605 + 0.930605i$
$(-3)^{1/5}$	1/5	≈ -1.245731	$\approx 1.007818 + 0.732222i$
$(-3)^{1/6}$	1/6	No real value	$\approx 1.040042 + 0.600468i$
$(-3)^{1/7}$	1/7	≈ -1.169931	$\approx 1.054071 + 0.507614i$
$(-3)^{1/8}$	1/8	No real value	$\approx 1.059877 + 0.439015i$
$(-3)^{1/9}$	1/9	≈ -1.129831	$\approx 1.061694 + 0.386425i$
$(-3)^{1/10}$	1/10	No real value	$\approx 1.061496 + 0.344901i$
$(-3)^{1/1}$	1	$= -3$	$= -3$
$(-3)^{2/1}$	2	$= 9$	$= 9$
$(-3)^{2/2}$	1	$= -3$	$= -3$
$(-3)^{2/3}$	2/3	≈ 2.080084	$\approx -1.040042 + 1.801405i$
$(-3)^{2/4}$	1/2	No real value	$\approx 1.732051i$
$(-3)^{2/5}$	2/5	≈ 1.551846	$\approx 0.479547 + 1.475893i$
$(-3)^{2/6}$	1/3	≈ -1.442250	$\approx 0.721125 + 1.249025i$
$(-3)^{2/7}$	2/7	≈ 1.368738	$\approx 0.853394 + 1.070123i$
$(-3)^{2/8}$	1/4	No real value	$\approx 0.930605 + 0.930605i$
$(-3)^{2/9}$	2/9	≈ 1.276518	$\approx 0.977870 + 0.820530i$
$(-3)^{2/10}$	1/5	≈ -1.245731	$\approx 1.007818 + 0.732222i$
$(-3)^{3/1}$	3	$= -27$	$= -27$
$(-3)^{3/2}$	3/2	No real value	$\approx -5.196152i$
$(-3)^{3/3}$	1	$= -3$	$= -3$
$(-3)^{3/4}$	3/4	No real value	$\approx -1.611855 + 1.611855i$
$(-3)^{3/5}$	3/5	≈ -1.933182	$\approx -0.597386 + 1.838565i$
$(-3)^{3/6}$	1/2	No real value	$\approx 1.732051i$
$(-3)^{3/7}$	3/7	≈ -1.601329	$\approx 0.356329 + 1.561180i$
$(-3)^{3/8}$	3/8	No real value	$\approx 0.577777 + 1.394877i$
$(-3)^{3/9}$	1/3	≈ -1.442250	$\approx 0.721125 + 1.249025i$
$(-3)^{3/10}$	3/10	No real value	$\approx 0.817250 + 1.124848i$
$(-3)^{4/1}$	4	$= 81$	$= 81$
$(-3)^{4/2}$	2	$= 9$	$= 9$
$(-3)^{4/3}$	4/3	≈ 4.326749	$\approx -2.163374 - 3.747074i$
$(-3)^{4/4}$	1	$= -3$	$= -3$

Exact definition: $P = 3^{r/s}[\cos(\pi r/s) + i \sin(\pi r/s)]$. A nonreal principal value can coexist with a real odd-root value.

Base -3: real & principal complex

Reduce p/q to r/s. Odd s permits a real value; even s does not. The principal complex column uses $\text{Arg}(-3) = \pi$.

TERM	r/s	REAL VALUE	PRINCIPAL COMPLEX VALUE
$(-3)^{4/5}$	4/5	≈ 2.408225	$\approx -1.948295 + 1.415519i$
$(-3)^{4/6}$	2/3	≈ 2.080084	$\approx -1.040042 + 1.801405i$
$(-3)^{4/7}$	4/7	≈ 1.873444	$\approx -0.416881 + 1.826473i$
$(-3)^{4/8}$	1/2	No real value	$\approx 1.732051i$
$(-3)^{4/9}$	4/9	≈ 1.629498	$\approx 0.282959 + 1.604742i$
$(-3)^{4/10}$	2/5	≈ 1.551846	$\approx 0.479547 + 1.475893i$
$(-3)^{5/1}$	5	$= -243$	$= -243$
$(-3)^{5/2}$	5/2	No real value	$\approx 15.588457i$
$(-3)^{5/3}$	5/3	≈ -6.240251	$\approx 3.120126 - 5.404216i$
$(-3)^{5/4}$	5/4	No real value	$\approx -2.791815 - 2.791815i$
$(-3)^{5/5}$	1	$= -3$	$= -3$
$(-3)^{5/6}$	5/6	No real value	$\approx -2.163374 + 1.249025i$
$(-3)^{5/7}$	5/7	≈ -2.191800	$\approx -1.366565 + 1.713618i$
$(-3)^{5/8}$	5/8	No real value	$\approx -0.760397 + 1.835761i$
$(-3)^{5/9}$	5/9	≈ -1.841058	$\approx -0.319696 + 1.813088i$
$(-3)^{5/10}$	1/2	No real value	$\approx 1.732051i$
$(-3)^{6/1}$	6	$= 729$	$= 729$
$(-3)^{6/2}$	3	$= -27$	$= -27$
$(-3)^{6/3}$	2	$= 9$	$= 9$
$(-3)^{6/4}$	3/2	No real value	$\approx -5.196152i$
$(-3)^{6/5}$	6/5	≈ 3.737193	$\approx -3.023453 - 2.196667i$
$(-3)^{6/6}$	1	$= -3$	$= -3$
$(-3)^{6/7}$	6/7	≈ 2.564254	$\approx -2.310313 + 1.112588i$
$(-3)^{6/8}$	3/4	No real value	$\approx -1.611855 + 1.611855i$
$(-3)^{6/9}$	2/3	≈ 2.080084	$\approx -1.040042 + 1.801405i$
$(-3)^{6/10}$	3/5	≈ -1.933182	$\approx -0.597386 + 1.838565i$
$(-3)^{7/1}$	7	$= -2,187$	$= -2,187$
$(-3)^{7/2}$	7/2	No real value	$\approx -46.765372i$
$(-3)^{7/3}$	7/3	≈ -12.980246	$\approx 6.490123 + 11.241223i$
$(-3)^{7/4}$	7/4	No real value	$\approx 4.835565 - 4.835565i$
$(-3)^{7/5}$	7/5	≈ -4.655537	$\approx -1.438640 - 4.427679i$
$(-3)^{7/6}$	7/6	No real value	$\approx -3.120126 - 1.801405i$
$(-3)^{7/7}$	1	$= -3$	$= -3$

The real column uses $(-1)^r \times 3^{r/s}$ when s is odd. Even reduced numerators give positive real values; odd ones give negative real values.

Base -3: real & principal complex

Reduce p/q to r/s. Odd s permits a real value; even s does not. The principal complex column uses $\text{Arg}(-3) = \pi$.

TERM	r/s	REAL VALUE	PRINCIPAL COMPLEX VALUE
$(-3)^{7/8}$	7/8	No real value	$\approx -2.415997 + 1.000739i$
$(-3)^{7/9}$	7/9	≈ -2.350143	$\approx -1.800314 + 1.510643i$
$(-3)^{7/10}$	7/10	No real value	$\approx -1.268246 + 1.745591i$
$(-3)^{8/1}$	8	$= 6,561$	$= 6,561$
$(-3)^{8/2}$	4	$= 81$	$= 81$
$(-3)^{8/3}$	8/3	≈ 18.720754	$\approx -9.360377 + 16.212649i$
$(-3)^{8/4}$	2	$= 9$	$= 9$
$(-3)^{8/5}$	8/5	≈ 5.799546	$\approx 1.792158 - 5.515696i$
$(-3)^{8/6}$	4/3	≈ 4.326749	$\approx -2.163374 - 3.747074i$
$(-3)^{8/7}$	8/7	≈ 3.509792	$\approx -3.162214 - 1.522842i$
$(-3)^{8/8}$	1	$= -3$	$= -3$
$(-3)^{8/9}$	8/9	≈ 2.655264	$\approx -2.495132 + 0.908154i$
$(-3)^{8/10}$	4/5	≈ 2.408225	$\approx -1.948295 + 1.415519i$
$(-3)^{9/1}$	9	$= -19,683$	$= -19,683$
$(-3)^{9/2}$	9/2	No real value	$\approx 140.296115i$
$(-3)^{9/3}$	3	$= -27$	$= -27$
$(-3)^{9/4}$	9/4	No real value	$\approx 8.375444 + 8.375444i$
$(-3)^{9/5}$	9/5	≈ -7.224674	$\approx 5.844884 - 4.246557i$
$(-3)^{9/6}$	3/2	No real value	$\approx -5.196152i$
$(-3)^{9/7}$	9/7	≈ -4.106214	$\approx -2.560183 - 3.210368i$
$(-3)^{9/8}$	9/8	No real value	$\approx -3.179631 - 1.317046i$
$(-3)^{9/9}$	1	$= -3$	$= -3$
$(-3)^{9/10}$	9/10	No real value	$\approx -2.556321 + 0.830599i$
$(-3)^{10/1}$	10	$= 59,049$	$= 59,049$
$(-3)^{10/2}$	5	$= -243$	$= -243$
$(-3)^{10/3}$	10/3	≈ 38.940738	$\approx -19.470369 - 33.723669i$
$(-3)^{10/4}$	5/2	No real value	$\approx 15.588457i$
$(-3)^{10/5}$	2	$= 9$	$= 9$
$(-3)^{10/6}$	5/3	≈ -6.240251	$\approx 3.120126 - 5.404216i$
$(-3)^{10/7}$	10/7	≈ 4.803987	$\approx -1.068988 - 4.683541i$
$(-3)^{10/8}$	5/4	No real value	$\approx -2.791815 - 2.791815i$
$(-3)^{10/10}$	1	$= -3$	$= -3$
$(-3)^{10/9}$	10/9	≈ 3.389493	$\approx -3.185081 - 1.159275i$

Equivalent fractions give the same value within each convention. Principal complex values agree with real values at integer exponents; noninteger powers with odd reduced denominators select different branches.

Base 3: complete exponent table

Positive real convention. All decimal values marked $\approx$ are rounded to six places. Read down the left block, then down the right block.

TERM	r/s	VALUE	TERM	r/s	VALUE
$3^{1/2}$	1/2	≈ 1.732051	$3^{3/6}$	1/2	≈ 1.732051
$3^{1/3}$	1/3	≈ 1.442250	$3^{3/7}$	3/7	≈ 1.601329
$3^{1/4}$	1/4	≈ 1.316074	$3^{3/8}$	3/8	≈ 1.509804
$3^{1/5}$	1/5	≈ 1.245731	$3^{3/9}$	1/3	≈ 1.442250
$3^{1/6}$	1/6	≈ 1.200937	$3^{3/10}$	3/10	≈ 1.390389
$3^{1/7}$	1/7	≈ 1.169931	$3^{4/1}$	4	$= 81$
$3^{1/8}$	1/8	≈ 1.147203	$3^{4/2}$	2	$= 9$
$3^{1/9}$	1/9	≈ 1.129831	$3^{4/3}$	4/3	≈ 4.326749
$3^{1/10}$	1/10	≈ 1.116123	$3^{4/4}$	1	$= 3$
$3^{1/1}$	1	$= 3$	$3^{4/5}$	4/5	≈ 2.408225
$3^{2/1}$	2	$= 9$	$3^{4/6}$	2/3	≈ 2.080084
$3^{2/2}$	1	$= 3$	$3^{4/7}$	4/7	≈ 1.873444
$3^{2/3}$	2/3	≈ 2.080084	$3^{4/8}$	1/2	≈ 1.732051
$3^{2/4}$	1/2	≈ 1.732051	$3^{4/9}$	4/9	≈ 1.629498
$3^{2/5}$	2/5	≈ 1.551846	$3^{4/10}$	2/5	≈ 1.551846
$3^{2/6}$	1/3	≈ 1.442250	$3^{5/1}$	5	$= 243$
$3^{2/7}$	2/7	≈ 1.368738	$3^{5/2}$	5/2	≈ 15.588457
$3^{2/8}$	1/4	≈ 1.316074	$3^{5/3}$	5/3	≈ 6.240251
$3^{2/9}$	2/9	≈ 1.276518	$3^{5/4}$	5/4	≈ 3.948222
$3^{2/10}$	1/5	≈ 1.245731	$3^{5/5}$	1	$= 3$
$3^{3/1}$	3	$= 27$	$3^{5/6}$	5/6	≈ 2.498050
$3^{3/2}$	3/2	≈ 5.196152	$3^{5/7}$	5/7	≈ 2.191800
$3^{3/3}$	1	$= 3$	$3^{5/8}$	5/8	≈ 1.987013
$3^{3/4}$	3/4	≈ 2.279507	$3^{5/9}$	5/9	≈ 1.841058
$3^{3/5}$	3/5	≈ 1.933182	$3^{5/10}$	1/2	≈ 1.732051

PATTERN TO NOTICE

The base 3 is prime. Equivalent fractions give identical values even when their unreduced numerators and denominators differ.

Base 3: complete exponent table

Positive real convention. All decimal values marked $\approx$ are rounded to six places. Read down the left block, then down the right block.

TERM	r/s	VALUE
$3^{6/1}$	6	= 729
$3^{6/2}$	3	= 27
$3^{6/3}$	2	= 9
$3^{6/4}$	$3/2$	≈ 5.196152
$3^{6/5}$	$6/5$	≈ 3.737193
$3^{6/6}$	1	= 3
$3^{6/7}$	$6/7$	≈ 2.564254
$3^{6/8}$	$3/4$	≈ 2.279507
$3^{6/9}$	$2/3$	≈ 2.080084
$3^{6/10}$	$3/5$	≈ 1.933182
$3^{7/1}$	7	= 2,187
$3^{7/2}$	$7/2$	≈ 46.765372
$3^{7/3}$	$7/3$	≈ 12.980246
$3^{7/4}$	$7/4$	≈ 6.838521
$3^{7/5}$	$7/5$	≈ 4.655537
$3^{7/6}$	$7/6$	≈ 3.602811
$3^{7/7}$	1	= 3
$3^{7/8}$	$7/8$	≈ 2.615057
$3^{7/9}$	$7/9$	≈ 2.350143
$3^{7/10}$	$7/10$	≈ 2.157669
$3^{8/1}$	8	= 6,561
$3^{8/2}$	4	= 81
$3^{8/3}$	$8/3$	≈ 18.720754
$3^{8/4}$	2	= 9
$3^{8/5}$	$8/5$	≈ 5.799546

TERM	r/s	VALUE
$3^{8/6}$	$4/3$	≈ 4.326749
$3^{8/7}$	$8/7$	≈ 3.509792
$3^{8/8}$	1	= 3
$3^{8/9}$	$8/9$	≈ 2.655264
$3^{8/10}$	$4/5$	≈ 2.408225
$3^{9/1}$	9	= 19,683
$3^{9/2}$	$9/2$	≈ 140.296115
$3^{9/3}$	3	= 27
$3^{9/4}$	$9/4$	≈ 11.844666
$3^{9/5}$	$9/5$	≈ 7.224674
$3^{9/6}$	$3/2$	≈ 5.196152
$3^{9/7}$	$9/7$	≈ 4.106214
$3^{9/8}$	$9/8$	≈ 3.441608
$3^{9/9}$	1	= 3
$3^{9/10}$	$9/10$	≈ 2.687875
$3^{10/1}$	10	= 59,049
$3^{10/2}$	5	= 243
$3^{10/3}$	$10/3$	≈ 38.940738
$3^{10/4}$	$5/2$	≈ 15.588457
$3^{10/5}$	2	= 9
$3^{10/6}$	$5/3$	≈ 6.240251
$3^{10/7}$	$10/7$	≈ 4.803987
$3^{10/8}$	$5/4$	≈ 3.948222
$3^{10/10}$	1	= 3
$3^{10/9}$	$10/9$	≈ 3.389493

PATTERN TO NOTICE

The base 3 is prime. Equivalent fractions give identical values even when their unreduced numerators and denominators differ.

Base -4: real & principal complex

Reduce p/q to r/s. Odd s permits a real value; even s does not. The principal complex column uses $\text{Arg}(-4) = \pi$.

TERM	r/s	REAL VALUE	PRINCIPAL COMPLEX VALUE
$(-4)^{1/2}$	1/2	No real value	$= 2i$
$(-4)^{1/3}$	1/3	≈ -1.587401	$\approx 0.793701 + 1.374730i$
$(-4)^{1/4}$	1/4	No real value	$\approx 1.000000 + 1.000000i$
$(-4)^{1/5}$	1/5	≈ -1.319508	$\approx 1.067504 + 0.775587i$
$(-4)^{1/6}$	1/6	No real value	$\approx 1.091124 + 0.629961i$
$(-4)^{1/7}$	1/7	≈ -1.219014	$\approx 1.098293 + 0.528910i$
$(-4)^{1/8}$	1/8	No real value	$\approx 1.098684 + 0.455090i$
$(-4)^{1/9}$	1/9	≈ -1.166529	$\approx 1.096179 + 0.398976i$
$(-4)^{1/10}$	1/10	No real value	$\approx 1.092477 + 0.354967i$
$(-4)^{1/1}$	1	$= -4$	$= -4$
$(-4)^{2/1}$	2	$= 16$	$= 16$
$(-4)^{2/2}$	1	$= -4$	$= -4$
$(-4)^{2/3}$	2/3	≈ 2.519842	$\approx -1.259921 + 2.182247i$
$(-4)^{2/4}$	1/2	No real value	$= 2i$
$(-4)^{2/5}$	2/5	≈ 1.741101	$\approx 0.538030 + 1.655886i$
$(-4)^{2/6}$	1/3	≈ -1.587401	$\approx 0.793701 + 1.374730i$
$(-4)^{2/7}$	2/7	≈ 1.485994	$\approx 0.926502 + 1.161797i$
$(-4)^{2/8}$	1/4	No real value	$\approx 1.000000 + 1.000000i$
$(-4)^{2/9}$	2/9	≈ 1.360790	$\approx 1.042426 + 0.874699i$
$(-4)^{2/10}$	1/5	≈ -1.319508	$\approx 1.067504 + 0.775587i$
$(-4)^{3/1}$	3	$= -64$	$= -64$
$(-4)^{3/2}$	3/2	No real value	$= -8i$
$(-4)^{3/3}$	1	$= -4$	$= -4$
$(-4)^{3/4}$	3/4	No real value	$\approx -2.000000 + 2.000000i$
$(-4)^{3/5}$	3/5	≈ -2.297397	$\approx -0.709935 + 2.184954i$
$(-4)^{3/6}$	1/2	No real value	$= 2i$
$(-4)^{3/7}$	3/7	≈ -1.811447	$\approx 0.403085 + 1.766031i$
$(-4)^{3/8}$	3/8	No real value	$\approx 0.643594 + 1.553774i$
$(-4)^{3/9}$	1/3	≈ -1.587401	$\approx 0.793701 + 1.374730i$
$(-4)^{3/10}$	3/10	No real value	$\approx 0.890916 + 1.226240i$
$(-4)^{4/1}$	4	$= 256$	$= 256$
$(-4)^{4/2}$	2	$= 16$	$= 16$
$(-4)^{4/3}$	4/3	≈ 6.349604	$\approx -3.174802 - 5.498919i$
$(-4)^{4/4}$	1	$= -4$	$= -4$

Exact definition: $P = 4^{r/s} [\cos(\pi r/s) + i \sin(\pi r/s)]$. A nonreal principal value can coexist with a real odd-root value.

Base -4: real & principal complex

Reduce p/q to r/s. Odd s permits a real value; even s does not. The principal complex column uses $\text{Arg}(-4) = \pi$.

TERM	r/s	REAL VALUE	PRINCIPAL COMPLEX VALUE
$(-4)^{4/5}$	4/5	≈ 3.031433	$\approx -2.452481 + 1.781832i$
$(-4)^{4/6}$	2/3	≈ 2.519842	$\approx -1.259921 + 2.182247i$
$(-4)^{4/7}$	4/7	≈ 2.208179	$\approx -0.491366 + 2.152815i$
$(-4)^{4/8}$	1/2	No real value	$= 2i$
$(-4)^{4/9}$	4/9	≈ 1.851749	$\approx 0.321553 + 1.823617i$
$(-4)^{4/10}$	2/5	≈ 1.741101	$\approx 0.538030 + 1.655886i$
$(-4)^{5/1}$	5	$= -1,024$	$= -1,024$
$(-4)^{5/2}$	5/2	No real value	$= 32i$
$(-4)^{5/3}$	5/3	≈ -10.079368	$\approx 5.039684 - 8.728989i$
$(-4)^{5/4}$	5/4	No real value	$\approx -4.000000 - 4.000000i$
$(-4)^{5/5}$	1	$= -4$	$= -4$
$(-4)^{5/6}$	5/6	No real value	$\approx -2.749459 + 1.587401i$
$(-4)^{5/7}$	5/7	≈ -2.691800	$\approx -1.678310 + 2.104534i$
$(-4)^{5/8}$	5/8	No real value	$\approx -0.910180 + 2.197368i$
$(-4)^{5/9}$	5/9	≈ -2.160119	$\approx -0.375101 + 2.127302i$
$(-4)^{5/10}$	1/2	No real value	$= 2i$
$(-4)^{6/1}$	6	$= 4,096$	$= 4,096$
$(-4)^{6/2}$	3	$= -64$	$= -64$
$(-4)^{6/3}$	2	$= 16$	$= 16$
$(-4)^{6/4}$	3/2	No real value	$= -8i$
$(-4)^{6/5}$	6/5	≈ 5.278032	$\approx -4.270017 - 3.102349i$
$(-4)^{6/6}$	1	$= -4$	$= -4$
$(-4)^{6/7}$	6/7	≈ 3.281341	$\approx -2.956386 + 1.423721i$
$(-4)^{6/8}$	3/4	No real value	$\approx -2.000000 + 2.000000i$
$(-4)^{6/9}$	2/3	≈ 2.519842	$\approx -1.259921 + 2.182247i$
$(-4)^{6/10}$	3/5	≈ -2.297397	$\approx -0.709935 + 2.184954i$
$(-4)^{7/1}$	7	$= -16,384$	$= -16,384$
$(-4)^{7/2}$	7/2	No real value	$= -128i$
$(-4)^{7/3}$	7/3	≈ -25.398417	$\approx 12.699208 + 21.995674i$
$(-4)^{7/4}$	7/4	No real value	$\approx 8.000000 - 8.000000i$
$(-4)^{7/5}$	7/5	≈ -6.964405	$\approx -2.152119 - 6.623542i$
$(-4)^{7/6}$	7/6	No real value	$\approx -4.364495 - 2.519842i$
$(-4)^{7/7}$	1	$= -4$	$= -4$

The real column uses $(-1)^r \times 4^{r/s}$ when s is odd. Even reduced numerators give positive real values; odd ones give negative real values.

Base -4: real & principal complex

Reduce p/q to r/s. Odd s permits a real value; even s does not. The principal complex column uses $\text{Arg}(-4) = \pi$.

TERM	r/s	REAL VALUE	PRINCIPAL COMPLEX VALUE
$(-4)^{7/8}$	7/8	No real value	$\approx -3.107548 + 1.287189i$
$(-4)^{7/9}$	7/9	≈ -2.939469	$\approx -2.251764 + 1.889454i$
$(-4)^{7/10}$	7/10	No real value	$\approx -1.551175 + 2.135009i$
$(-4)^{8/1}$	8	$= 65,536$	$= 65,536$
$(-4)^{8/2}$	4	$= 256$	$= 256$
$(-4)^{8/3}$	8/3	≈ 40.317474	$\approx -20.158737 + 34.915956i$
$(-4)^{8/4}$	2	$= 16$	$= 16$
$(-4)^{8/5}$	8/5	≈ 9.189587	$\approx 2.839739 - 8.739816i$
$(-4)^{8/6}$	4/3	≈ 6.349604	$\approx -3.174802 - 5.498919i$
$(-4)^{8/7}$	8/7	≈ 4.876055	$\approx -4.393173 - 2.115641i$
$(-4)^{8/8}$	1	$= -4$	$= -4$
$(-4)^{8/9}$	8/9	≈ 3.428976	$\approx -3.222183 + 1.172779i$
$(-4)^{8/10}$	4/5	≈ 3.031433	$\approx -2.452481 + 1.781832i$
$(-4)^{9/1}$	9	$= -262,144$	$= -262,144$
$(-4)^{9/2}$	9/2	No real value	$= 512i$
$(-4)^{9/3}$	3	$= -64$	$= -64$
$(-4)^{9/4}$	9/4	No real value	$\approx 16.000000 + 16.000000i$
$(-4)^{9/5}$	9/5	≈ -12.125733	$\approx 9.809924 - 7.127327i$
$(-4)^{9/6}$	3/2	No real value	$= -8i$
$(-4)^{9/7}$	9/7	≈ -5.943977	$\approx -3.706009 - 4.647188i$
$(-4)^{9/8}$	9/8	No real value	$\approx -4.394736 - 1.820359i$
$(-4)^{9/9}$	1	$= -4$	$= -4$
$(-4)^{9/10}$	9/10	No real value	$\approx -3.311771 + 1.076060i$
$(-4)^{10/1}$	10	$= 1,048,576$	$= 1,048,576$
$(-4)^{10/2}$	5	$= -1,024$	$= -1,024$
$(-4)^{10/3}$	10/3	≈ 101.593667	$\approx -50.796834 - 87.982697i$
$(-4)^{10/4}$	5/2	No real value	$= 32i$
$(-4)^{10/5}$	2	$= 16$	$= 16$
$(-4)^{10/6}$	5/3	≈ -10.079368	$\approx 5.039684 - 8.728989i$
$(-4)^{10/7}$	10/7	≈ 7.245789	$\approx -1.612340 - 7.064122i$
$(-4)^{10/8}$	5/4	No real value	$\approx -4.000000 - 4.000000i$
$(-4)^{10/10}$	1	$= -4$	$= -4$
$(-4)^{10/9}$	10/9	≈ 4.666116	$\approx -4.384715 - 1.595906i$

Equivalent fractions give the same value within each convention. Principal complex values agree with real values at integer exponents; noninteger powers with odd reduced denominators select different branches.

Base 4: complete exponent table

Positive real convention. All decimal values marked $\approx$ are rounded to six places. Read down the left block, then down the right block.

TERM	r/s	VALUE
$4^{1/2}$	1/2	= 2
$4^{1/3}$	1/3	≈ 1.587401
$4^{1/4}$	1/4	≈ 1.414214
$4^{1/5}$	1/5	≈ 1.319508
$4^{1/6}$	1/6	≈ 1.259921
$4^{1/7}$	1/7	≈ 1.219014
$4^{1/8}$	1/8	≈ 1.189207
$4^{1/9}$	1/9	≈ 1.166529
$4^{1/10}$	1/10	≈ 1.148698
$4^{1/1}$	1	= 4
$4^{2/1}$	2	= 16
$4^{2/2}$	1	= 4
$4^{2/3}$	2/3	≈ 2.519842
$4^{2/4}$	1/2	= 2
$4^{2/5}$	2/5	≈ 1.741101
$4^{2/6}$	1/3	≈ 1.587401
$4^{2/7}$	2/7	≈ 1.485994
$4^{2/8}$	1/4	≈ 1.414214
$4^{2/9}$	2/9	≈ 1.360790
$4^{2/10}$	1/5	≈ 1.319508
$4^{3/1}$	3	= 64
$4^{3/2}$	3/2	= 8
$4^{3/3}$	1	= 4
$4^{3/4}$	3/4	≈ 2.828427
$4^{3/5}$	3/5	≈ 2.297397

TERM	r/s	VALUE
$4^{3/6}$	1/2	= 2
$4^{3/7}$	3/7	≈ 1.811447
$4^{3/8}$	3/8	≈ 1.681793
$4^{3/9}$	1/3	≈ 1.587401
$4^{3/10}$	3/10	≈ 1.515717
$4^{4/1}$	4	= 256
$4^{4/2}$	2	= 16
$4^{4/3}$	4/3	≈ 6.349604
$4^{4/4}$	1	= 4
$4^{4/5}$	4/5	≈ 3.031433
$4^{4/6}$	2/3	≈ 2.519842
$4^{4/7}$	4/7	≈ 2.208179
$4^{4/8}$	1/2	= 2
$4^{4/9}$	4/9	≈ 1.851749
$4^{4/10}$	2/5	≈ 1.741101
$4^{5/1}$	5	= 1,024
$4^{5/2}$	5/2	= 32
$4^{5/3}$	5/3	≈ 10.079368
$4^{5/4}$	5/4	≈ 5.656854
$4^{5/5}$	1	= 4
$4^{5/6}$	5/6	≈ 3.174802
$4^{5/7}$	5/7	≈ 2.691800
$4^{5/8}$	5/8	≈ 2.378414
$4^{5/9}$	5/9	≈ 2.160119
$4^{5/10}$	1/2	= 2

PATTERN TO NOTICE

Since $4 = 2^2$, reduced denominators 1 and 2 give integer results. Example: $4^{3/2} = 8$.

Base 4: complete exponent table

Positive real convention. All decimal values marked $\approx$ are rounded to six places. Read down the left block, then down the right block.

TERM	r/s	VALUE
$4^{6/1}$	6	= 4,096
$4^{6/2}$	3	= 64
$4^{6/3}$	2	= 16
$4^{6/4}$	$3/2$	= 8
$4^{6/5}$	$6/5$	≈ 5.278032
$4^{6/6}$	1	= 4
$4^{6/7}$	$6/7$	≈ 3.281341
$4^{6/8}$	$3/4$	≈ 2.828427
$4^{6/9}$	$2/3$	≈ 2.519842
$4^{6/10}$	$3/5$	≈ 2.297397
$4^{7/1}$	7	= 16,384
$4^{7/2}$	$7/2$	= 128
$4^{7/3}$	$7/3$	≈ 25.398417
$4^{7/4}$	$7/4$	≈ 11.313708
$4^{7/5}$	$7/5$	≈ 6.964405
$4^{7/6}$	$7/6$	≈ 5.039684
$4^{7/7}$	1	= 4
$4^{7/8}$	$7/8$	≈ 3.363586
$4^{7/9}$	$7/9$	≈ 2.939469
$4^{7/10}$	$7/10$	≈ 2.639016
$4^{8/1}$	8	= 65,536
$4^{8/2}$	4	= 256
$4^{8/3}$	$8/3$	≈ 40.317474
$4^{8/4}$	2	= 16
$4^{8/5}$	$8/5$	≈ 9.189587

TERM	r/s	VALUE
$4^{8/6}$	$4/3$	≈ 6.349604
$4^{8/7}$	$8/7$	≈ 4.876055
$4^{8/8}$	1	= 4
$4^{8/9}$	$8/9$	≈ 3.428976
$4^{8/10}$	$4/5$	≈ 3.031433
$4^{9/1}$	9	= 262,144
$4^{9/2}$	$9/2$	= 512
$4^{9/3}$	3	= 64
$4^{9/4}$	$9/4$	≈ 22.627417
$4^{9/5}$	$9/5$	≈ 12.125733
$4^{9/6}$	$3/2$	= 8
$4^{9/7}$	$9/7$	≈ 5.943977
$4^{9/8}$	$9/8$	≈ 4.756828
$4^{9/9}$	1	= 4
$4^{9/10}$	$9/10$	≈ 3.482202
$4^{10/1}$	10	= 1,048,576
$4^{10/2}$	5	= 1,024
$4^{10/3}$	$10/3$	≈ 101.593667
$4^{10/4}$	$5/2$	= 32
$4^{10/5}$	2	= 16
$4^{10/6}$	$5/3$	≈ 10.079368
$4^{10/7}$	$10/7$	≈ 7.245789
$4^{10/8}$	$5/4$	≈ 5.656854
$4^{10/10}$	1	= 4
$4^{10/9}$	$10/9$	≈ 4.666116

PATTERN TO NOTICE

Since $4 = 2^2$, reduced denominators 1 and 2 give integer results. Example: $4^{3/2} = 8$.

Base -5: real & principal complex

Reduce p/q to r/s. Odd s permits a real value; even s does not. The principal complex column uses $\text{Arg}(-5) = \pi$.

TERM	r/s	REAL VALUE	PRINCIPAL COMPLEX VALUE
$(-5)^{1/2}$	1/2	No real value	$\approx 2.236068i$
$(-5)^{1/3}$	1/3	≈ -1.709976	$\approx 0.854988 + 1.480883i$
$(-5)^{1/4}$	1/4	No real value	$\approx 1.057371 + 1.057371i$
$(-5)^{1/5}$	1/5	≈ -1.379730	$\approx 1.116225 + 0.810985i$
$(-5)^{1/6}$	1/6	No real value	$\approx 1.132467 + 0.653830i$
$(-5)^{1/7}$	1/7	≈ -1.258499	$\approx 1.133868 + 0.546042i$
$(-5)^{1/8}$	1/8	No real value	$\approx 1.129761 + 0.467962i$
$(-5)^{1/9}$	1/9	≈ -1.195813	$\approx 1.123697 + 0.408992i$
$(-5)^{1/10}$	1/10	No real value	$\approx 1.117129 + 0.362977i$
$(-5)^{1/1}$	1	= -5	= -5
$(-5)^{2/1}$	2	= 25	= 25
$(-5)^{2/2}$	1	= -5	= -5
$(-5)^{2/3}$	2/3	≈ 2.924018	$\approx -1.462009 + 2.532274i$
$(-5)^{2/4}$	1/2	No real value	$\approx 2.236068i$
$(-5)^{2/5}$	2/5	≈ 1.903654	$\approx 0.588261 + 1.810482i$
$(-5)^{2/6}$	1/3	≈ -1.709976	$\approx 0.854988 + 1.480883i$
$(-5)^{2/7}$	2/7	≈ 1.583820	$\approx 0.987495 + 1.238280i$
$(-5)^{2/8}$	1/4	No real value	$\approx 1.057371 + 1.057371i$
$(-5)^{2/9}$	2/9	≈ 1.429969	$\approx 1.095420 + 0.919166i$
$(-5)^{2/10}$	1/5	≈ -1.379730	$\approx 1.116225 + 0.810985i$
$(-5)^{3/1}$	3	= -125	= -125
$(-5)^{3/2}$	3/2	No real value	$\approx -11.180340i$
$(-5)^{3/3}$	1	= -5	= -5
$(-5)^{3/4}$	3/4	No real value	$\approx -2.364354 + 2.364354i$
$(-5)^{3/5}$	3/5	≈ -2.626528	$\approx -0.811642 + 2.497976i$
$(-5)^{3/6}$	1/2	No real value	$\approx 2.236068i$
$(-5)^{3/7}$	3/7	≈ -1.993235	$\approx 0.443537 + 1.943261i$
$(-5)^{3/8}$	3/8	No real value	$\approx 0.699767 + 1.689387i$
$(-5)^{3/9}$	1/3	≈ -1.709976	$\approx 0.854988 + 1.480883i$
$(-5)^{3/10}$	3/10	No real value	$\approx 0.952598 + 1.311139i$
$(-5)^{4/1}$	4	= 625	= 625
$(-5)^{4/2}$	2	= 25	= 25
$(-5)^{4/3}$	4/3	≈ 8.549880	$\approx -4.274940 - 7.404413i$
$(-5)^{4/4}$	1	= -5	= -5

Exact definition: $P = 5^{r/s} [\cos(\pi r/s) + i \sin(\pi r/s)]$. A nonreal principal value can coexist with a real odd-root value.

Base -5: real & principal complex

Reduce p/q to r/s. Odd s permits a real value; even s does not. The principal complex column uses $\text{Arg}(-5) = \pi$.

TERM	r/s	REAL VALUE	PRINCIPAL COMPLEX VALUE
$(-5)^{4/5}$	4/5	≈ 3.623898	$\approx -2.931795 + 2.130074i$
$(-5)^{4/6}$	2/3	≈ 2.924018	$\approx -1.462009 + 2.532274i$
$(-5)^{4/7}$	4/7	≈ 2.508485	$\approx -0.558190 + 2.445592i$
$(-5)^{4/8}$	1/2	No real value	$\approx 2.236068i$
$(-5)^{4/9}$	4/9	≈ 2.044812	$\approx 0.355078 + 2.013746i$
$(-5)^{4/10}$	2/5	≈ 1.903654	$\approx 0.588261 + 1.810482i$
$(-5)^{5/1}$	5	$= -3,125$	$= -3,125$
$(-5)^{5/2}$	5/2	No real value	$\approx 55.901699i$
$(-5)^{5/3}$	5/3	≈ -14.620089	$\approx 7.310044 - 12.661368i$
$(-5)^{5/4}$	5/4	No real value	$\approx -5.286856 - 5.286856i$
$(-5)^{5/5}$	1	$= -5$	$= -5$
$(-5)^{5/6}$	5/6	No real value	$\approx -3.311354 + 1.911811i$
$(-5)^{5/7}$	5/7	≈ -3.156925	$\approx -1.968311 + 2.468183i$
$(-5)^{5/8}$	5/8	No real value	$\approx -1.046396 + 2.526222i$
$(-5)^{5/9}$	5/9	≈ -2.445213	$\approx -0.424607 + 2.408065i$
$(-5)^{5/10}$	1/2	No real value	$\approx 2.236068i$
$(-5)^{6/1}$	6	$= 15,625$	$= 15,625$
$(-5)^{6/2}$	3	$= -125$	$= -125$
$(-5)^{6/3}$	2	$= 25$	$= 25$
$(-5)^{6/4}$	3/2	No real value	$\approx -11.180340i$
$(-5)^{6/5}$	6/5	≈ 6.898648	$\approx -5.581124 - 4.054924i$
$(-5)^{6/6}$	1	$= -5$	$= -5$
$(-5)^{6/7}$	6/7	≈ 3.972987	$\approx -3.579538 + 1.723814i$
$(-5)^{6/8}$	3/4	No real value	$\approx -2.364354 + 2.364354i$
$(-5)^{6/9}$	2/3	≈ 2.924018	$\approx -1.462009 + 2.532274i$
$(-5)^{6/10}$	3/5	≈ -2.626528	$\approx -0.811642 + 2.497976i$
$(-5)^{7/1}$	7	$= -78,125$	$= -78,125$
$(-5)^{7/2}$	7/2	No real value	$\approx -279.508497i$
$(-5)^{7/3}$	7/3	≈ -42.749399	$\approx 21.374699 + 37.022065i$
$(-5)^{7/4}$	7/4	No real value	$\approx 11.821770 - 11.821770i$
$(-5)^{7/5}$	7/5	≈ -9.518270	$\approx -2.941307 - 9.052412i$
$(-5)^{7/6}$	7/6	No real value	$\approx -5.662336 - 3.269151i$
$(-5)^{7/7}$	1	$= -5$	$= -5$

The real column uses $(-1)^r \times 5^{r/s}$ when s is odd. Even reduced numerators give positive real values; odd ones give negative real values.

Base -5: real & principal complex

Reduce p/q to r/s. Odd s permits a real value; even s does not. The principal complex column uses $\text{Arg}(-5) = \pi$.

TERM	r/s	REAL VALUE	PRINCIPAL COMPLEX VALUE
$(-5)^{7/8}$	7/8	No real value	$\approx -3.777584 + 1.564726i$
$(-5)^{7/9}$	7/9	≈ -3.496579	$\approx -2.678535 + 2.247558i$
$(-5)^{7/10}$	7/10	No real value	$\approx -1.813417 + 2.495954i$
$(-5)^{8/1}$	8	$= 390,625$	$= 390,625$
$(-5)^{8/2}$	4	$= 625$	$= 625$
$(-5)^{8/3}$	8/3	≈ 73.100443	$\approx -36.550222 + 63.306841i$
$(-5)^{8/4}$	2	$= 25$	$= 25$
$(-5)^{8/5}$	8/5	≈ 13.132639	$\approx 4.058209 - 12.489882i$
$(-5)^{8/6}$	4/3	≈ 8.549880	$\approx -4.274940 - 7.404413i$
$(-5)^{8/7}$	8/7	≈ 6.292495	$\approx -5.669342 - 2.730211i$
$(-5)^{8/8}$	1	$= -5$	$= -5$
$(-5)^{8/9}$	8/9	≈ 4.181255	$\approx -3.929095 + 1.430073i$
$(-5)^{8/10}$	4/5	≈ 3.623898	$\approx -2.931795 + 2.130074i$
$(-5)^{9/1}$	9	$= -1,953,125$	$= -1,953,125$
$(-5)^{9/2}$	9/2	No real value	$\approx 1,397.542486i$
$(-5)^{9/3}$	3	$= -125$	$= -125$
$(-5)^{9/4}$	9/4	No real value	$\approx 26.434282 + 26.434282i$
$(-5)^{9/5}$	9/5	≈ -18.119492	$\approx 14.658977 - 10.650370i$
$(-5)^{9/6}$	3/2	No real value	$\approx -11.180340i$
$(-5)^{9/7}$	9/7	≈ -7.919098	$\approx -4.937477 - 6.191400i$
$(-5)^{9/8}$	9/8	No real value	$\approx -5.648805 - 2.339812i$
$(-5)^{9/9}$	1	$= -5$	$= -5$
$(-5)^{9/10}$	9/10	No real value	$\approx -4.048362 + 1.315393i$
$(-5)^{10/1}$	10	$= 9,765,625$	$= 9,765,625$
$(-5)^{10/2}$	5	$= -3,125$	$= -3,125$
$(-5)^{10/3}$	10/3	≈ 213.746993	$\approx -106.873497 - 185.110326i$
$(-5)^{10/4}$	5/2	No real value	$\approx 55.901699i$
$(-5)^{10/5}$	2	$= 25$	$= 25$
$(-5)^{10/6}$	5/3	≈ -14.620089	$\approx 7.310044 - 12.661368i$
$(-5)^{10/7}$	10/7	≈ 9.966177	$\approx -2.217683 - 9.716304i$
$(-5)^{10/8}$	5/4	No real value	$\approx -5.286856 - 5.286856i$
$(-5)^{10/10}$	1	$= -5$	$= -5$
$(-5)^{10/9}$	10/9	≈ 5.979066	$\approx -5.618484 - 2.044961i$

Equivalent fractions give the same value within each convention. Principal complex values agree with real values at integer exponents; noninteger powers with odd reduced denominators select different branches.

Base 5: complete exponent table

Positive real convention. All decimal values marked $\approx$ are rounded to six places. Read down the left block, then down the right block.

TERM	r/s	VALUE	TERM	r/s	VALUE
$5^{1/2}$	1/2	≈ 2.236068	$5^{3/6}$	1/2	≈ 2.236068
$5^{1/3}$	1/3	≈ 1.709976	$5^{3/7}$	3/7	≈ 1.993235
$5^{1/4}$	1/4	≈ 1.495349	$5^{3/8}$	3/8	≈ 1.828579
$5^{1/5}$	1/5	≈ 1.379730	$5^{3/9}$	1/3	≈ 1.709976
$5^{1/6}$	1/6	≈ 1.307660	$5^{3/10}$	3/10	≈ 1.620657
$5^{1/7}$	1/7	≈ 1.258499	$5^{4/1}$	4	$= 625$
$5^{1/8}$	1/8	≈ 1.222845	$5^{4/2}$	2	$= 25$
$5^{1/9}$	1/9	≈ 1.195813	$5^{4/3}$	4/3	≈ 8.549880
$5^{1/10}$	1/10	≈ 1.174619	$5^{4/4}$	1	$= 5$
$5^{1/1}$	1	$= 5$	$5^{4/5}$	4/5	≈ 3.623898
$5^{2/1}$	2	$= 25$	$5^{4/6}$	2/3	≈ 2.924018
$5^{2/2}$	1	$= 5$	$5^{4/7}$	4/7	≈ 2.508485
$5^{2/3}$	2/3	≈ 2.924018	$5^{4/8}$	1/2	≈ 2.236068
$5^{2/4}$	1/2	≈ 2.236068	$5^{4/9}$	4/9	≈ 2.044812
$5^{2/5}$	2/5	≈ 1.903654	$5^{4/10}$	2/5	≈ 1.903654
$5^{2/6}$	1/3	≈ 1.709976	$5^{5/1}$	5	$= 3,125$
$5^{2/7}$	2/7	≈ 1.583820	$5^{5/2}$	5/2	≈ 55.901699
$5^{2/8}$	1/4	≈ 1.495349	$5^{5/3}$	5/3	≈ 14.620089
$5^{2/9}$	2/9	≈ 1.429969	$5^{5/4}$	5/4	≈ 7.476744
$5^{2/10}$	1/5	≈ 1.379730	$5^{5/5}$	1	$= 5$
$5^{3/1}$	3	$= 125$	$5^{5/6}$	5/6	≈ 3.823622
$5^{3/2}$	3/2	≈ 11.180340	$5^{5/7}$	5/7	≈ 3.156925
$5^{3/3}$	1	$= 5$	$5^{5/8}$	5/8	≈ 2.734364
$5^{3/4}$	3/4	≈ 3.343702	$5^{5/9}$	5/9	≈ 2.445213
$5^{3/5}$	3/5	≈ 2.626528	$5^{5/10}$	1/2	≈ 2.236068

PATTERN TO NOTICE

The base 5 is prime. Compare a root exponent below 1 with an improper exponent above 1 to see how the value crosses 5.

Base 5: complete exponent table

Positive real convention. All decimal values marked $\approx$ are rounded to six places. Read down the left block, then down the right block.

TERM	r/s	VALUE
$5^{6/1}$	6	= 15,625
$5^{6/2}$	3	= 125
$5^{6/3}$	2	= 25
$5^{6/4}$	$3/2$	≈ 11.180340
$5^{6/5}$	$6/5$	≈ 6.898648
$5^{6/6}$	1	= 5
$5^{6/7}$	$6/7$	≈ 3.972987
$5^{6/8}$	$3/4$	≈ 3.343702
$5^{6/9}$	$2/3$	≈ 2.924018
$5^{6/10}$	$3/5$	≈ 2.626528
$5^{7/1}$	7	= 78,125
$5^{7/2}$	$7/2$	≈ 279.508497
$5^{7/3}$	$7/3$	≈ 42.749399
$5^{7/4}$	$7/4$	≈ 16.718508
$5^{7/5}$	$7/5$	≈ 9.518270
$5^{7/6}$	$7/6$	≈ 6.538302
$5^{7/7}$	1	= 5
$5^{7/8}$	$7/8$	≈ 4.088827
$5^{7/9}$	$7/9$	≈ 3.496579
$5^{7/10}$	$7/10$	≈ 3.085169
$5^{8/1}$	8	= 390,625
$5^{8/2}$	4	= 625
$5^{8/3}$	$8/3$	≈ 73.100443
$5^{8/4}$	2	= 25
$5^{8/5}$	$8/5$	≈ 13.132639

TERM	r/s	VALUE
$5^{8/6}$	$4/3$	≈ 8.549880
$5^{8/7}$	$8/7$	≈ 6.292495
$5^{8/8}$	1	= 5
$5^{8/9}$	$8/9$	≈ 4.181255
$5^{8/10}$	$4/5$	≈ 3.623898
$5^{9/1}$	9	= 1,953,125
$5^{9/2}$	$9/2$	$\approx 1,397.542486$
$5^{9/3}$	3	= 125
$5^{9/4}$	$9/4$	≈ 37.383720
$5^{9/5}$	$9/5$	≈ 18.119492
$5^{9/6}$	$3/2$	≈ 11.180340
$5^{9/7}$	$9/7$	≈ 7.919098
$5^{9/8}$	$9/8$	≈ 6.114223
$5^{9/9}$	1	= 5
$5^{9/10}$	$9/10$	≈ 4.256700
$5^{10/1}$	10	= 9,765,625
$5^{10/2}$	5	= 3,125
$5^{10/3}$	$10/3$	≈ 213.746993
$5^{10/4}$	$5/2$	≈ 55.901699
$5^{10/5}$	2	= 25
$5^{10/6}$	$5/3$	≈ 14.620089
$5^{10/7}$	$10/7$	≈ 9.966177
$5^{10/8}$	$5/4$	≈ 7.476744
$5^{10/10}$	1	= 5
$5^{10/9}$	$10/9$	≈ 5.979066

PATTERN TO NOTICE

The base 5 is prime. Compare a root exponent below 1 with an improper exponent above 1 to see how the value crosses 5.

Base -6: real & principal complex

Reduce p/q to r/s. Odd s permits a real value; even s does not. The principal complex column uses $\text{Arg}(-6) = \pi$.

TERM	r/s	REAL VALUE	PRINCIPAL COMPLEX VALUE
$(-6)^{1/2}$	1/2	No real value	$\approx 2.449490i$
$(-6)^{1/3}$	1/3	≈ -1.817121	$\approx 0.908560 + 1.573673i$
$(-6)^{1/4}$	1/4	No real value	$\approx 1.106682 + 1.106682i$
$(-6)^{1/5}$	1/5	≈ -1.430969	$\approx 1.157678 + 0.841103i$
$(-6)^{1/6}$	1/6	No real value	$\approx 1.167408 + 0.674003i$
$(-6)^{1/7}$	1/7	≈ -1.291708	$\approx 1.163789 + 0.560451i$
$(-6)^{1/8}$	1/8	No real value	$\approx 1.155804 + 0.478750i$
$(-6)^{1/9}$	1/9	≈ -1.220285	$\approx 1.146693 + 0.417362i$
$(-6)^{1/10}$	1/10	No real value	$\approx 1.137683 + 0.369656i$
$(-6)^{1/1}$	1	$= -6$	$= -6$
$(-6)^{2/1}$	2	$= 36$	$= 36$
$(-6)^{2/2}$	1	$= -6$	$= -6$
$(-6)^{2/3}$	2/3	≈ 3.301927	$\approx -1.650964 + 2.859553i$
$(-6)^{2/4}$	1/2	No real value	$\approx 2.449490i$
$(-6)^{2/5}$	2/5	≈ 2.047673	$\approx 0.632766 + 1.947452i$
$(-6)^{2/6}$	1/3	≈ -1.817121	$\approx 0.908560 + 1.573673i$
$(-6)^{2/7}$	2/7	≈ 1.668510	$\approx 1.040299 + 1.304494i$
$(-6)^{2/8}$	1/4	No real value	$\approx 1.106682 + 1.106682i$
$(-6)^{2/9}$	2/9	≈ 1.489095	$\approx 1.140713 + 0.957172i$
$(-6)^{2/10}$	1/5	≈ -1.430969	$\approx 1.157678 + 0.841103i$
$(-6)^{3/1}$	3	$= -216$	$= -216$
$(-6)^{3/2}$	3/2	No real value	$\approx -14.696938i$
$(-6)^{3/3}$	1	$= -6$	$= -6$
$(-6)^{3/4}$	3/4	No real value	$\approx -2.710806 + 2.710806i$
$(-6)^{3/5}$	3/5	≈ -2.930156	$\approx -0.905468 + 2.786744i$
$(-6)^{3/6}$	1/2	No real value	$\approx 2.449490i$
$(-6)^{3/7}$	3/7	≈ -2.155229	$\approx 0.479584 + 2.101193i$
$(-6)^{3/8}$	3/8	No real value	$\approx 0.749284 + 1.808931i$
$(-6)^{3/9}$	1/3	≈ -1.817121	$\approx 0.908560 + 1.573673i$
$(-6)^{3/10}$	3/10	No real value	$\approx 1.006153 + 1.384851i$
$(-6)^{4/1}$	4	$= 1,296$	$= 1,296$
$(-6)^{4/2}$	2	$= 36$	$= 36$
$(-6)^{4/3}$	4/3	≈ 10.902724	$\approx -5.451362 - 9.442036i$
$(-6)^{4/4}$	1	$= -6$	$= -6$

Exact definition: $P = 6^{r/s}[\cos(\pi r/s) + i \sin(\pi r/s)]$. A nonreal principal value can coexist with a real odd-root value.

Base -6: real & principal complex

Reduce p/q to r/s. Odd s permits a real value; even s does not. The principal complex column uses $\text{Arg}(-6) = \pi$.

TERM	r/s	REAL VALUE	PRINCIPAL COMPLEX VALUE
$(-6)^{4/5}$	4/5	≈ 4.192963	$\approx -3.392178 + 2.464562i$
$(-6)^{4/6}$	2/3	≈ 3.301927	$\approx -1.650964 + 2.859553i$
$(-6)^{4/7}$	4/7	≈ 2.783927	$\approx -0.619482 + 2.714128i$
$(-6)^{4/8}$	1/2	No real value	$\approx 2.449490i$
$(-6)^{4/9}$	4/9	≈ 2.217405	$\approx 0.385048 + 2.183718i$
$(-6)^{4/10}$	2/5	≈ 2.047673	$\approx 0.632766 + 1.947452i$
$(-6)^{5/1}$	5	$= -7,776$	$= -7,776$
$(-6)^{5/2}$	5/2	No real value	$\approx 88.181631i$
$(-6)^{5/3}$	5/3	≈ -19.811563	$\approx 9.905782 - 17.157317i$
$(-6)^{5/4}$	5/4	No real value	$\approx -6.640092 - 6.640092i$
$(-6)^{5/5}$	1	$= -6$	$= -6$
$(-6)^{5/6}$	5/6	No real value	$\approx -3.854695 + 2.225509i$
$(-6)^{5/7}$	5/7	≈ -3.596022	$\approx -2.242083 + 2.811483i$
$(-6)^{5/8}$	5/8	No real value	$\approx -1.172693 + 2.831130i$
$(-6)^{5/9}$	5/9	≈ -2.705866	$\approx -0.469869 + 2.664758i$
$(-6)^{5/10}$	1/2	No real value	$\approx 2.449490i$
$(-6)^{6/1}$	6	$= 46,656$	$= 46,656$
$(-6)^{6/2}$	3	$= -216$	$= -216$
$(-6)^{6/3}$	2	$= 36$	$= 36$
$(-6)^{6/4}$	3/2	No real value	$\approx -14.696938i$
$(-6)^{6/5}$	6/5	≈ 8.585814	$\approx -6.946070 - 5.046615i$
$(-6)^{6/6}$	1	$= -6$	$= -6$
$(-6)^{6/7}$	6/7	≈ 4.645011	$\approx -4.185011 + 2.015395i$
$(-6)^{6/8}$	3/4	No real value	$\approx -2.710806 + 2.710806i$
$(-6)^{6/9}$	2/3	≈ 3.301927	$\approx -1.650964 + 2.859553i$
$(-6)^{6/10}$	3/5	≈ -2.930156	$\approx -0.905468 + 2.786744i$
$(-6)^{7/1}$	7	$= -279,936$	$= -279,936$
$(-6)^{7/2}$	7/2	No real value	$\approx -529.089784i$
$(-6)^{7/3}$	7/3	≈ -65.416341	$\approx 32.708171 + 56.652213i$
$(-6)^{7/4}$	7/4	No real value	$\approx 16.264836 - 16.264836i$
$(-6)^{7/5}$	7/5	≈ -12.286035	$\approx -3.796594 - 11.684714i$
$(-6)^{7/6}$	7/6	No real value	$\approx -7.004445 - 4.044018i$
$(-6)^{7/7}$	1	$= -6$	$= -6$

The real column uses $(-1)^r \times 6^{r/s}$ when s is odd. Even reduced numerators give positive real values; odd ones give negative real values.

Base -6: real & principal complex

Reduce p/q to r/s. Odd s permits a real value; even s does not. The principal complex column uses $\text{Arg}(-6) = \pi$.

TERM	r/s	REAL VALUE	PRINCIPAL COMPLEX VALUE
$(-6)^{7/8}$	7/8	No real value	$\approx -4.430959 + 1.835363i$
$(-6)^{7/9}$	7/9	≈ -4.029292	$\approx -3.086617 + 2.589979i$
$(-6)^{7/10}$	7/10	No real value	$\approx -2.060272 + 2.835721i$
$(-6)^{8/1}$	8	$= 1,679,616$	$= 1,679,616$
$(-6)^{8/2}$	4	$= 1,296$	$= 1,296$
$(-6)^{8/3}$	8/3	≈ 118.869381	$\approx -59.434690 + 102.943904i$
$(-6)^{8/4}$	2	$= 36$	$= 36$
$(-6)^{8/5}$	8/5	≈ 17.580936	$\approx 5.432808 - 16.720464i$
$(-6)^{8/6}$	4/3	≈ 10.902724	$\approx -5.451362 - 9.442036i$
$(-6)^{8/7}$	8/7	≈ 7.750250	$\approx -6.982734 - 3.362707i$
$(-6)^{8/8}$	1	$= -6$	$= -6$
$(-6)^{8/9}$	8/9	≈ 4.916884	$\approx -4.620360 + 1.681674i$
$(-6)^{8/10}$	4/5	≈ 4.192963	$\approx -3.392178 + 2.464562i$
$(-6)^{9/1}$	9	$= -10,077,696$	$= -10,077,696$
$(-6)^{9/2}$	9/2	No real value	$\approx 3,174.538707i$
$(-6)^{9/3}$	3	$= -216$	$= -216$
$(-6)^{9/4}$	9/4	No real value	$\approx 39.840549 + 39.840549i$
$(-6)^{9/5}$	9/5	≈ -25.157776	$\approx 20.353069 - 14.787370i$
$(-6)^{9/6}$	3/2	No real value	$\approx -14.696938i$
$(-6)^{9/7}$	9/7	≈ -10.011063	$\approx -6.241795 - 7.826964i$
$(-6)^{9/8}$	9/8	No real value	$\approx -6.934825 - 2.872499i$
$(-6)^{9/9}$	1	$= -6$	$= -6$
$(-6)^{9/10}$	9/10	No real value	$\approx -4.770264 + 1.549953i$
$(-6)^{10/1}$	10	$= 60,466,176$	$= 60,466,176$
$(-6)^{10/2}$	5	$= -7,776$	$= -7,776$
$(-6)^{10/3}$	10/3	≈ 392.498048	$\approx -196.249024 - 339.913281i$
$(-6)^{10/4}$	5/2	No real value	$\approx 88.181631i$
$(-6)^{10/5}$	2	$= 36$	$= 36$
$(-6)^{10/6}$	5/3	≈ -19.811563	$\approx 9.905782 - 17.157317i$
$(-6)^{10/7}$	10/7	≈ 12.931373	$\approx -2.877501 - 12.607157i$
$(-6)^{10/8}$	5/4	No real value	$\approx -6.640092 - 6.640092i$
$(-6)^{10/10}$	1	$= -6$	$= -6$
$(-6)^{10/9}$	10/9	≈ 7.321710	$\approx -6.880156 - 2.504172i$

Equivalent fractions give the same value within each convention. Principal complex values agree with real values at integer exponents; noninteger powers with odd reduced denominators select different branches.

Base 6: complete exponent table

Positive real convention. All decimal values marked $\approx$ are rounded to six places. Read down the left block, then down the right block.

TERM	r/s	VALUE	TERM	r/s	VALUE
$6^{1/2}$	1/2	≈ 2.449490	$6^{3/6}$	1/2	≈ 2.449490
$6^{1/3}$	1/3	≈ 1.817121	$6^{3/7}$	3/7	≈ 2.155229
$6^{1/4}$	1/4	≈ 1.565085	$6^{3/8}$	3/8	≈ 1.957973
$6^{1/5}$	1/5	≈ 1.430969	$6^{3/9}$	1/3	≈ 1.817121
$6^{1/6}$	1/6	≈ 1.348006	$6^{3/10}$	3/10	≈ 1.711770
$6^{1/7}$	1/7	≈ 1.291708	$6^{4/1}$	4	$= 1,296$
$6^{1/8}$	1/8	≈ 1.251033	$6^{4/2}$	2	$= 36$
$6^{1/9}$	1/9	≈ 1.220285	$6^{4/3}$	4/3	≈ 10.902724
$6^{1/10}$	1/10	≈ 1.196231	$6^{4/4}$	1	$= 6$
$6^{1/1}$	1	$= 6$	$6^{4/5}$	4/5	≈ 4.192963
$6^{2/1}$	2	$= 36$	$6^{4/6}$	2/3	≈ 3.301927
$6^{2/2}$	1	$= 6$	$6^{4/7}$	4/7	≈ 2.783927
$6^{2/3}$	2/3	≈ 3.301927	$6^{4/8}$	1/2	≈ 2.449490
$6^{2/4}$	1/2	≈ 2.449490	$6^{4/9}$	4/9	≈ 2.217405
$6^{2/5}$	2/5	≈ 2.047673	$6^{4/10}$	2/5	≈ 2.047673
$6^{2/6}$	1/3	≈ 1.817121	$6^{5/1}$	5	$= 7,776$
$6^{2/7}$	2/7	≈ 1.668510	$6^{5/2}$	5/2	≈ 88.181631
$6^{2/8}$	1/4	≈ 1.565085	$6^{5/3}$	5/3	≈ 19.811563
$6^{2/9}$	2/9	≈ 1.489095	$6^{5/4}$	5/4	≈ 9.390507
$6^{2/10}$	1/5	≈ 1.430969	$6^{5/5}$	1	$= 6$
$6^{3/1}$	3	$= 216$	$6^{5/6}$	5/6	≈ 4.451018
$6^{3/2}$	3/2	≈ 14.696938	$6^{5/7}$	5/7	≈ 3.596022
$6^{3/3}$	1	$= 6$	$6^{5/8}$	5/8	≈ 3.064393
$6^{3/4}$	3/4	≈ 3.833659	$6^{5/9}$	5/9	≈ 2.705866
$6^{3/5}$	3/5	≈ 2.930156	$6^{5/10}$	1/2	≈ 2.449490

PATTERN TO NOTICE

Since $6 = 2 \times 3$, a noninteger reduced exponent gives an irrational value. Both prime exponents must pass the perfect-power test.

Base 6: complete exponent table

Positive real convention. All decimal values marked $\approx$ are rounded to six places. Read down the left block, then down the right block.

TERM	r/s	VALUE
$6^{6/1}$	6	= 46,656
$6^{6/2}$	3	= 216
$6^{6/3}$	2	= 36
$6^{6/4}$	$3/2$	≈ 14.696938
$6^{6/5}$	$6/5$	≈ 8.585814
$6^{6/6}$	1	= 6
$6^{6/7}$	$6/7$	≈ 4.645011
$6^{6/8}$	$3/4$	≈ 3.833659
$6^{6/9}$	$2/3$	≈ 3.301927
$6^{6/10}$	$3/5$	≈ 2.930156
$6^{7/1}$	7	= 279,936
$6^{7/2}$	$7/2$	≈ 529.089784
$6^{7/3}$	$7/3$	≈ 65.416341
$6^{7/4}$	$7/4$	≈ 23.001952
$6^{7/5}$	$7/5$	≈ 12.286035
$6^{7/6}$	$7/6$	≈ 8.088037
$6^{7/7}$	1	= 6
$6^{7/8}$	$7/8$	≈ 4.796035
$6^{7/9}$	$7/9$	≈ 4.029292
$6^{7/10}$	$7/10$	≈ 3.505144
$6^{8/1}$	8	= 1,679,616
$6^{8/2}$	4	= 1,296
$6^{8/3}$	$8/3$	≈ 118.869381
$6^{8/4}$	2	= 36
$6^{8/5}$	$8/5$	≈ 17.580936

TERM	r/s	VALUE
$6^{8/6}$	$4/3$	≈ 10.902724
$6^{8/7}$	$8/7$	≈ 7.750250
$6^{8/8}$	1	= 6
$6^{8/9}$	$8/9$	≈ 4.916884
$6^{8/10}$	$4/5$	≈ 4.192963
$6^{9/1}$	9	= 10,077,696
$6^{9/2}$	$9/2$	$\approx 3,174.538707$
$6^{9/3}$	3	= 216
$6^{9/4}$	$9/4$	≈ 56.343045
$6^{9/5}$	$9/5$	≈ 25.157776
$6^{9/6}$	$3/2$	≈ 14.696938
$6^{9/7}$	$9/7$	≈ 10.011063
$6^{9/8}$	$9/8$	≈ 7.506200
$6^{9/9}$	1	= 6
$6^{9/10}$	$9/10$	≈ 5.015753
$6^{10/1}$	10	= 60,466,176
$6^{10/2}$	5	= 7,776
$6^{10/3}$	$10/3$	≈ 392.498048
$6^{10/4}$	$5/2$	≈ 88.181631
$6^{10/5}$	2	= 36
$6^{10/6}$	$5/3$	≈ 19.811563
$6^{10/7}$	$10/7$	≈ 12.931373
$6^{10/8}$	$5/4$	≈ 9.390507
$6^{10/10}$	1	= 6
$6^{10/9}$	$10/9$	≈ 7.321710

PATTERN TO NOTICE

Since $6 = 2 \times 3$, a noninteger reduced exponent gives an irrational value. Both prime exponents must pass the perfect-power test.

Base -7: real & principal complex

Reduce p/q to r/s. Odd s permits a real value; even s does not. The principal complex column uses $\text{Arg}(-7) = \pi$.

TERM	r/s	REAL VALUE	PRINCIPAL COMPLEX VALUE
$(-7)^{1/2}$	1/2	No real value	$\approx 2.645751i$
$(-7)^{1/3}$	1/3	≈ -1.912931	$\approx 0.956466 + 1.656647i$
$(-7)^{1/4}$	1/4	No real value	$\approx 1.150163 + 1.150163i$
$(-7)^{1/5}$	1/5	≈ -1.475773	$\approx 1.193926 + 0.867438i$
$(-7)^{1/6}$	1/6	No real value	$\approx 1.197789 + 0.691544i$
$(-7)^{1/7}$	1/7	≈ -1.320469	$\approx 1.189702 + 0.572930i$
$(-7)^{1/8}$	1/8	No real value	$\approx 1.178291 + 0.488064i$
$(-7)^{1/9}$	1/9	≈ -1.241366	$\approx 1.166502 + 0.424572i$
$(-7)^{1/10}$	1/10	No real value	$\approx 1.155357 + 0.375398i$
$(-7)^{1/1}$	1	$= -7$	$= -7$
$(-7)^{2/1}$	2	$= 49$	$= 49$
$(-7)^{2/2}$	1	$= -7$	$= -7$
$(-7)^{2/3}$	2/3	≈ 3.659306	$\approx -1.829653 + 3.169052i$
$(-7)^{2/4}$	1/2	No real value	$\approx 2.645751i$
$(-7)^{2/5}$	2/5	≈ 2.177906	$\approx 0.673010 + 2.071312i$
$(-7)^{2/6}$	1/3	≈ -1.912931	$\approx 0.956466 + 1.656647i$
$(-7)^{2/7}$	2/7	≈ 1.743639	$\approx 1.087141 + 1.363232i$
$(-7)^{2/8}$	1/4	No real value	$\approx 1.150163 + 1.150163i$
$(-7)^{2/9}$	2/9	≈ 1.540989	$\approx 1.180466 + 0.990529i$
$(-7)^{2/10}$	1/5	≈ -1.475773	$\approx 1.193926 + 0.867438i$
$(-7)^{3/1}$	3	$= -343$	$= -343$
$(-7)^{3/2}$	3/2	No real value	$\approx -18.520259i$
$(-7)^{3/3}$	1	$= -7$	$= -7$
$(-7)^{3/4}$	3/4	No real value	$\approx -3.043046 + 3.043046i$
$(-7)^{3/5}$	3/5	≈ -3.214096	$\approx -0.993210 + 3.056787i$
$(-7)^{3/6}$	1/2	No real value	$\approx 2.645751i$
$(-7)^{3/7}$	3/7	≈ -2.302422	$\approx 0.512337 + 2.244695i$
$(-7)^{3/8}$	3/8	No real value	$\approx 0.793874 + 1.916581i$
$(-7)^{3/9}$	1/3	≈ -1.912931	$\approx 0.956466 + 1.656647i$
$(-7)^{3/10}$	3/10	No real value	$\approx 1.053776 + 1.450398i$
$(-7)^{4/1}$	4	$= 2,401$	$= 2,401$
$(-7)^{4/2}$	2	$= 49$	$= 49$
$(-7)^{4/3}$	4/3	≈ 13.390518	$\approx -6.695259 - 11.596529i$
$(-7)^{4/4}$	1	$= -7$	$= -7$

Exact definition: $P = 7^{r/s}[\cos(\pi r/s) + i \sin(\pi r/s)]$. A nonreal principal value can coexist with a real odd-root value.

Base -7: real & principal complex

Reduce p/q to r/s. Odd s permits a real value; even s does not. The principal complex column uses $\text{Arg}(-7) = \pi$.

TERM	r/s	REAL VALUE	PRINCIPAL COMPLEX VALUE
$(-7)^{4/5}$	4/5	≈ 4.743276	$\approx -3.837391 + 2.788028i$
$(-7)^{4/6}$	2/3	≈ 3.659306	$\approx -1.829653 + 3.169052i$
$(-7)^{4/7}$	4/7	≈ 3.040277	$\approx -0.676525 + 2.964051i$
$(-7)^{4/8}$	1/2	No real value	$\approx 2.645751i$
$(-7)^{4/9}$	4/9	≈ 2.374647	$\approx 0.412353 + 2.338571i$
$(-7)^{4/10}$	2/5	≈ 2.177906	$\approx 0.673010 + 2.071312i$
$(-7)^{5/1}$	5	$= -16,807$	$= -16,807$
$(-7)^{5/2}$	5/2	No real value	$\approx 129.641814i$
$(-7)^{5/3}$	5/3	≈ -25.615140	$\approx 12.807570 - 22.183362i$
$(-7)^{5/4}$	5/4	No real value	$\approx -8.051143 - 8.051143i$
$(-7)^{5/5}$	1	$= -7$	$= -7$
$(-7)^{5/6}$	5/6	No real value	$\approx -4.383076 + 2.530570i$
$(-7)^{5/7}$	5/7	≈ -4.014592	$\approx -2.503057 + 3.138735i$
$(-7)^{5/8}$	5/8	No real value	$\approx -1.291296 + 3.117465i$
$(-7)^{5/9}$	5/9	≈ -2.947806	$\approx -0.511881 + 2.903022i$
$(-7)^{5/10}$	1/2	No real value	$\approx 2.645751i$
$(-7)^{6/1}$	6	$= 117,649$	$= 117,649$
$(-7)^{6/2}$	3	$= -343$	$= -343$
$(-7)^{6/3}$	2	$= 49$	$= 49$
$(-7)^{6/4}$	3/2	No real value	$\approx -18.520259i$
$(-7)^{6/5}$	6/5	≈ 10.330412	$\approx -8.357479 - 6.072064i$
$(-7)^{6/6}$	1	$= -7$	$= -7$
$(-7)^{6/7}$	6/7	≈ 5.301146	$\approx -4.776167 + 2.300081i$
$(-7)^{6/8}$	3/4	No real value	$\approx -3.043046 + 3.043046i$
$(-7)^{6/9}$	2/3	≈ 3.659306	$\approx -1.829653 + 3.169052i$
$(-7)^{6/10}$	3/5	≈ -3.214096	$\approx -0.993210 + 3.056787i$
$(-7)^{7/1}$	7	$= -823,543$	$= -823,543$
$(-7)^{7/2}$	7/2	No real value	$\approx -907.492700i$
$(-7)^{7/3}$	7/3	≈ -93.733628	$\approx 46.866814 + 81.175703i$
$(-7)^{7/4}$	7/4	No real value	$\approx 21.301323 - 21.301323i$
$(-7)^{7/5}$	7/5	≈ -15.245345	$\approx -4.711071 - 14.499185i$
$(-7)^{7/6}$	7/6	No real value	$\approx -8.384523 - 4.840806i$
$(-7)^{7/7}$	1	$= -7$	$= -7$

The real column uses $(-1)^r \times 7^{r/s}$ when s is odd. Even reduced numerators give positive real values; odd ones give negative real values.

Base -7: real & principal complex

Reduce p/q to r/s. Odd s permits a real value; even s does not. The principal complex column uses $\text{Arg}(-7) = \pi$.

TERM	r/s	REAL VALUE	PRINCIPAL COMPLEX VALUE
$(-7)^{7/8}$	7/8	No real value	$\approx -5.070796 + 2.100392i$
$(-7)^{7/9}$	7/9	≈ -4.542537	$\approx -3.479785 + 2.919887i$
$(-7)^{7/10}$	7/10	No real value	$\approx -2.295024 + 3.158830i$
$(-7)^{8/1}$	8	$= 5,764,801$	$= 5,764,801$
$(-7)^{8/2}$	4	$= 2,401$	$= 2,401$
$(-7)^{8/3}$	8/3	≈ 179.305980	$\approx -89.652990 + 155.283534i$
$(-7)^{8/4}$	2	$= 49$	$= 49$
$(-7)^{8/5}$	8/5	≈ 22.498671	$\approx 6.952472 - 21.397508i$
$(-7)^{8/6}$	4/3	≈ 13.390518	$\approx -6.695259 - 11.596529i$
$(-7)^{8/7}$	8/7	≈ 9.243285	$\approx -8.327912 - 4.010511i$
$(-7)^{8/8}$	1	$= -7$	$= -7$
$(-7)^{8/9}$	8/9	≈ 5.638950	$\approx -5.298880 + 1.928635i$
$(-7)^{8/10}$	4/5	≈ 4.743276	$\approx -3.837391 + 2.788028i$
$(-7)^{9/1}$	9	$= -40,353,607$	$= -40,353,607$
$(-7)^{9/2}$	9/2	No real value	$\approx 6,352.448898i$
$(-7)^{9/3}$	3	$= -343$	$= -343$
$(-7)^{9/4}$	9/4	No real value	$\approx 56.358003 + 56.358003i$
$(-7)^{9/5}$	9/5	≈ -33.202935	$\approx 26.861738 - 19.516195i$
$(-7)^{9/6}$	3/2	No real value	$\approx -18.520259i$
$(-7)^{9/7}$	9/7	≈ -12.205473	$\approx -7.609988 - 9.542623i$
$(-7)^{9/8}$	9/8	No real value	$\approx -8.248038 - 3.416449i$
$(-7)^{9/9}$	1	$= -7$	$= -7$
$(-7)^{9/10}$	9/10	No real value	$\approx -5.480177 + 1.780617i$
$(-7)^{10/1}$	10	$= 282,475,249$	$= 282,475,249$
$(-7)^{10/2}$	5	$= -16,807$	$= -16,807$
$(-7)^{10/3}$	10/3	≈ 656.135396	$\approx -328.067698 - 568.229921i$
$(-7)^{10/4}$	5/2	No real value	$\approx 129.641814i$
$(-7)^{10/5}$	2	$= 49$	$= 49$
$(-7)^{10/6}$	5/3	≈ -25.615140	$\approx 12.807570 - 22.183362i$
$(-7)^{10/7}$	10/7	≈ 16.116952	$\approx -3.586359 - 15.712866i$
$(-7)^{10/8}$	5/4	No real value	$\approx -8.051143 - 8.051143i$
$(-7)^{10/10}$	1	$= -7$	$= -7$
$(-7)^{10/9}$	10/9	≈ 8.689561	$\approx -8.165516 - 2.972005i$

Equivalent fractions give the same value within each convention. Principal complex values agree with real values at integer exponents; noninteger powers with odd reduced denominators select different branches.

Base 7: complete exponent table

Positive real convention. All decimal values marked $\approx$ are rounded to six places. Read down the left block, then down the right block.

TERM	r/s	VALUE	TERM	r/s	VALUE
$7^{1/2}$	1/2	≈ 2.645751	$7^{3/6}$	1/2	≈ 2.645751
$7^{1/3}$	1/3	≈ 1.912931	$7^{3/7}$	3/7	≈ 2.302422
$7^{1/4}$	1/4	≈ 1.626577	$7^{3/8}$	3/8	≈ 2.074492
$7^{1/5}$	1/5	≈ 1.475773	$7^{3/9}$	1/3	≈ 1.912931
$7^{1/6}$	1/6	≈ 1.383088	$7^{3/10}$	3/10	≈ 1.792790
$7^{1/7}$	1/7	≈ 1.320469	$7^{4/1}$	4	$= 2,401$
$7^{1/8}$	1/8	≈ 1.275373	$7^{4/2}$	2	$= 49$
$7^{1/9}$	1/9	≈ 1.241366	$7^{4/3}$	4/3	≈ 13.390518
$7^{1/10}$	1/10	≈ 1.214814	$7^{4/4}$	1	$= 7$
$7^{1/1}$	1	$= 7$	$7^{4/5}$	4/5	≈ 4.743276
$7^{2/1}$	2	$= 49$	$7^{4/6}$	2/3	≈ 3.659306
$7^{2/2}$	1	$= 7$	$7^{4/7}$	4/7	≈ 3.040277
$7^{2/3}$	2/3	≈ 3.659306	$7^{4/8}$	1/2	≈ 2.645751
$7^{2/4}$	1/2	≈ 2.645751	$7^{4/9}$	4/9	≈ 2.374647
$7^{2/5}$	2/5	≈ 2.177906	$7^{4/10}$	2/5	≈ 2.177906
$7^{2/6}$	1/3	≈ 1.912931	$7^{5/1}$	5	$= 16,807$
$7^{2/7}$	2/7	≈ 1.743639	$7^{5/2}$	5/2	≈ 129.641814
$7^{2/8}$	1/4	≈ 1.626577	$7^{5/3}$	5/3	≈ 25.615140
$7^{2/9}$	2/9	≈ 1.540989	$7^{5/4}$	5/4	≈ 11.386036
$7^{2/10}$	1/5	≈ 1.475773	$7^{5/5}$	1	$= 7$
$7^{3/1}$	3	$= 343$	$7^{5/6}$	5/6	≈ 5.061140
$7^{3/2}$	3/2	≈ 18.520259	$7^{5/7}$	5/7	≈ 4.014592
$7^{3/3}$	1	$= 7$	$7^{5/8}$	5/8	≈ 3.374320
$7^{3/4}$	3/4	≈ 4.303517	$7^{5/9}$	5/9	≈ 2.947806
$7^{3/5}$	3/5	≈ 3.214096	$7^{5/10}$	1/2	≈ 2.645751

PATTERN TO NOTICE

The base 7 is prime. At a fixed positive base greater than 1, larger exponents produce larger values.

Base 7: complete exponent table

Positive real convention. All decimal values marked $\approx$ are rounded to six places. Read down the left block, then down the right block.

TERM	r/s	VALUE	TERM	r/s	VALUE
$7^{6/1}$	6	= 117,649	$7^{8/6}$	4/3	≈ 13.390518
$7^{6/2}$	3	= 343	$7^{8/7}$	8/7	≈ 9.243285
$7^{6/3}$	2	= 49	$7^{8/8}$	1	= 7
$7^{6/4}$	3/2	≈ 18.520259	$7^{8/9}$	8/9	≈ 5.638950
$7^{6/5}$	6/5	≈ 10.330412	$7^{8/10}$	4/5	≈ 4.743276
$7^{6/6}$	1	= 7	$7^{9/1}$	9	= 40,353,607
$7^{6/7}$	6/7	≈ 5.301146	$7^{9/2}$	9/2	$\approx 6,352.448898$
$7^{6/8}$	3/4	≈ 4.303517	$7^{9/3}$	3	= 343
$7^{6/9}$	2/3	≈ 3.659306	$7^{9/4}$	9/4	≈ 79.702252
$7^{6/10}$	3/5	≈ 3.214096	$7^{9/5}$	9/5	≈ 33.202935
$7^{7/1}$	7	= 823,543	$7^{9/6}$	3/2	≈ 18.520259
$7^{7/2}$	7/2	≈ 907.492700	$7^{9/7}$	9/7	≈ 12.205473
$7^{7/3}$	7/3	≈ 93.733628	$7^{9/8}$	9/8	≈ 8.927612
$7^{7/4}$	7/4	≈ 30.124619	$7^{9/9}$	1	= 7
$7^{7/5}$	7/5	≈ 15.245345	$7^{9/10}$	9/10	≈ 5.762199
$7^{7/6}$	7/6	≈ 9.681613	$7^{10/1}$	10	= 282,475,249
$7^{7/7}$	1	= 7	$7^{10/2}$	5	= 16,807
$7^{7/8}$	7/8	≈ 5.488590	$7^{10/3}$	10/3	≈ 656.135396
$7^{7/9}$	7/9	≈ 4.542537	$7^{10/4}$	5/2	≈ 129.641814
$7^{7/10}$	7/10	≈ 3.904529	$7^{10/5}$	2	= 49
$7^{8/1}$	8	= 5,764,801	$7^{10/6}$	5/3	≈ 25.615140
$7^{8/2}$	4	= 2,401	$7^{10/7}$	10/7	≈ 16.116952
$7^{8/3}$	8/3	≈ 179.305980	$7^{10/8}$	5/4	≈ 11.386036
$7^{8/4}$	2	= 49	$7^{10/10}$	1	= 7
$7^{8/5}$	8/5	≈ 22.498671	$7^{10/9}$	10/9	≈ 8.689561

PATTERN TO NOTICE

The base 7 is prime. At a fixed positive base greater than 1, larger exponents produce larger values.

Base -8: real & principal complex

Reduce p/q to r/s. Odd s permits a real value; even s does not. The principal complex column uses $\text{Arg}(-8) = \pi$.

TERM	r/s	REAL VALUE	PRINCIPAL COMPLEX VALUE
$(-8)^{1/2}$	1/2	No real value	$\approx 2.828427i$
$(-8)^{1/3}$	1/3	$= -2$	$\approx 1.000000 + 1.732051i$
$(-8)^{1/4}$	1/4	No real value	$\approx 1.189207 + 1.189207i$
$(-8)^{1/5}$	1/5	≈ -1.515717	$\approx 1.226240 + 0.890916i$
$(-8)^{1/6}$	1/6	No real value	$\approx 1.224745 + 0.707107i$
$(-8)^{1/7}$	1/7	≈ -1.345900	$\approx 1.212614 + 0.583964i$
$(-8)^{1/8}$	1/8	No real value	$\approx 1.198124 + 0.496279i$
$(-8)^{1/9}$	1/9	≈ -1.259921	$\approx 1.183939 + 0.430918i$
$(-8)^{1/10}$	1/10	No real value	$\approx 1.170888 + 0.380445i$
$(-8)^{1/1}$	1	$= -8$	$= -8$
$(-8)^{2/1}$	2	$= 64$	$= 64$
$(-8)^{2/2}$	1	$= -8$	$= -8$
$(-8)^{2/3}$	2/3	$= 4$	$\approx -2.000000 + 3.464102i$
$(-8)^{2/4}$	1/2	No real value	$\approx 2.828427i$
$(-8)^{2/5}$	2/5	≈ 2.297397	$\approx 0.709935 + 2.184954i$
$(-8)^{2/6}$	1/3	$= -2$	$\approx 1.000000 + 1.732051i$
$(-8)^{2/7}$	2/7	≈ 1.811447	$\approx 1.129419 + 1.416247i$
$(-8)^{2/8}$	1/4	No real value	$\approx 1.189207 + 1.189207i$
$(-8)^{2/9}$	2/9	≈ 1.587401	$\approx 1.216020 + 1.020362i$
$(-8)^{2/10}$	1/5	≈ -1.515717	$\approx 1.226240 + 0.890916i$
$(-8)^{3/1}$	3	$= -512$	$= -512$
$(-8)^{3/2}$	3/2	No real value	$\approx -22.627417i$
$(-8)^{3/3}$	1	$= -8$	$= -8$
$(-8)^{3/4}$	3/4	No real value	$\approx -3.363586 + 3.363586i$
$(-8)^{3/5}$	3/5	≈ -3.482202	$\approx -1.076060 + 3.311771i$
$(-8)^{3/6}$	1/2	No real value	$\approx 2.828427i$
$(-8)^{3/7}$	3/7	≈ -2.438027	$\approx 0.542512 + 2.376901i$
$(-8)^{3/8}$	3/8	No real value	$\approx 0.834638 + 2.014996i$
$(-8)^{3/9}$	1/3	$= -2$	$\approx 1.000000 + 1.732051i$
$(-8)^{3/10}$	3/10	No real value	$\approx 1.096846 + 1.509679i$
$(-8)^{4/1}$	4	$= 4,096$	$= 4,096$
$(-8)^{4/2}$	2	$= 64$	$= 64$
$(-8)^{4/3}$	4/3	$= 16$	$\approx -8.000000 - 13.856406i$
$(-8)^{4/4}$	1	$= -8$	$= -8$

Exact definition: $P = 8^{r/s} [\cos(\pi r/s) + i \sin(\pi r/s)]$. A nonreal principal value can coexist with a real odd-root value.

Base -8: real & principal complex

Reduce p/q to r/s. Odd s permits a real value; even s does not. The principal complex column uses $\text{Arg}(-8) = \pi$.

TERM	r/s	REAL VALUE	PRINCIPAL COMPLEX VALUE
$(-8)^{4/5}$	4/5	≈ 5.278032	$\approx -4.270017 + 3.102349i$
$(-8)^{4/6}$	2/3	$= 4$	$\approx -2.000000 + 3.464102i$
$(-8)^{4/7}$	4/7	≈ 3.281341	$\approx -0.730167 + 3.199071i$
$(-8)^{4/8}$	1/2	No real value	$\approx 2.828427i$
$(-8)^{4/9}$	4/9	≈ 2.519842	$\approx 0.437566 + 2.481560i$
$(-8)^{4/10}$	2/5	≈ 2.297397	$\approx 0.709935 + 2.184954i$
$(-8)^{5/1}$	5	$= -32,768$	$= -32,768$
$(-8)^{5/2}$	5/2	No real value	$\approx 181.019336i$
$(-8)^{5/3}$	5/3	$= -32$	$\approx 16.000000 - 27.712813i$
$(-8)^{5/4}$	5/4	No real value	$\approx -9.513657 - 9.513657i$
$(-8)^{5/5}$	1	$= -8$	$= -8$
$(-8)^{5/6}$	5/6	No real value	$\approx -4.898979 + 2.828427i$
$(-8)^{5/7}$	5/7	≈ -4.416358	$\approx -2.753554 + 3.452848i$
$(-8)^{5/8}$	5/8	No real value	$\approx -1.403689 + 3.388805i$
$(-8)^{5/9}$	5/9	≈ -3.174802	$\approx -0.551299 + 3.126570i$
$(-8)^{5/10}$	1/2	No real value	$\approx 2.828427i$
$(-8)^{6/1}$	6	$= 262,144$	$= 262,144$
$(-8)^{6/2}$	3	$= -512$	$= -512$
$(-8)^{6/3}$	2	$= 64$	$= 64$
$(-8)^{6/4}$	3/2	No real value	$\approx -22.627417i$
$(-8)^{6/5}$	6/5	≈ 12.125733	$\approx -9.809924 - 7.127327i$
$(-8)^{6/6}$	1	$= -8$	$= -8$
$(-8)^{6/7}$	6/7	≈ 5.943977	$\approx -5.355338 + 2.578995i$
$(-8)^{6/8}$	3/4	No real value	$\approx -3.363586 + 3.363586i$
$(-8)^{6/9}$	2/3	$= 4$	$\approx -2.000000 + 3.464102i$
$(-8)^{6/10}$	3/5	≈ -3.482202	$\approx -1.076060 + 3.311771i$
$(-8)^{7/1}$	7	$= -2,097,152$	$= -2,097,152$
$(-8)^{7/2}$	7/2	No real value	$\approx -1,448.154688i$
$(-8)^{7/3}$	7/3	$= -128$	$\approx 64.000000 + 110.851252i$
$(-8)^{7/4}$	7/4	No real value	$\approx 26.908685 - 26.908685i$
$(-8)^{7/5}$	7/5	≈ -18.379174	$\approx -5.679477 - 17.479633i$
$(-8)^{7/6}$	7/6	No real value	$\approx -9.797959 - 5.656854i$
$(-8)^{7/7}$	1	$= -8$	$= -8$

The real column uses $(-1)^r \times 8^{r/s}$ when s is odd. Even reduced numerators give positive real values; odd ones give negative real values.

Base -8: real & principal complex

Reduce p/q to r/s. Odd s permits a real value; even s does not. The principal complex column uses $\text{Arg}(-8) = \pi$.

TERM	r/s	REAL VALUE	PRINCIPAL COMPLEX VALUE
$(-8)^{7/8}$	7/8	No real value	$\approx -5.699268 + 2.360714i$
$(-8)^{7/9}$	7/9	≈ -5.039684	$\approx -3.860622 + 3.239447i$
$(-8)^{7/10}$	7/10	No real value	$\approx -2.519891 + 3.468332i$
$(-8)^{8/1}$	8	$= 16,777,216$	$= 16,777,216$
$(-8)^{8/2}$	4	$= 4,096$	$= 4,096$
$(-8)^{8/3}$	8/3	$= 256$	$\approx -128.000000 + 221.702503i$
$(-8)^{8/4}$	2	$= 64$	$= 64$
$(-8)^{8/5}$	8/5	≈ 27.857618	$\approx 8.608477 - 26.494169i$
$(-8)^{8/6}$	4/3	$= 16$	$\approx -8.000000 - 13.856406i$
$(-8)^{8/7}$	8/7	≈ 10.767202	$\approx -9.700913 - 4.671714i$
$(-8)^{8/8}$	1	$= -8$	$= -8$
$(-8)^{8/9}$	8/9	≈ 6.349604	$\approx -5.966676 + 2.171693i$
$(-8)^{8/10}$	4/5	≈ 5.278032	$\approx -4.270017 + 3.102349i$
$(-8)^{9/1}$	9	$= -134,217,728$	$= -134,217,728$
$(-8)^{9/2}$	9/2	No real value	$\approx 11,585.237503i$
$(-8)^{9/3}$	3	$= -512$	$= -512$
$(-8)^{9/4}$	9/4	No real value	$\approx 76.109255 + 76.109255i$
$(-8)^{9/5}$	9/5	≈ -42.224253	$\approx 34.160138 - 24.818793i$
$(-8)^{9/6}$	3/2	No real value	$\approx -22.627417i$
$(-8)^{9/7}$	9/7	≈ -14.491579	$\approx -9.035351 - 11.329972i$
$(-8)^{9/8}$	9/8	No real value	$\approx -9.584988 - 3.970232i$
$(-8)^{9/9}$	1	$= -8$	$= -8$
$(-8)^{9/10}$	9/10	No real value	$\approx -6.179983 + 2.007998i$
$(-8)^{10/1}$	10	$= 1,073,741,824$	$= 1,073,741,824$
$(-8)^{10/2}$	5	$= -32,768$	$= -32,768$
$(-8)^{10/3}$	10/3	$= 1,024$	$\approx -512.000000 - 886.810013i$
$(-8)^{10/4}$	5/2	No real value	$\approx 181.019336i$
$(-8)^{10/5}$	2	$= 64$	$= 64$
$(-8)^{10/6}$	5/3	$= -32$	$\approx 16.000000 - 27.712813i$
$(-8)^{10/7}$	10/7	≈ 19.504218	$\approx -4.340097 - 19.015207i$
$(-8)^{10/8}$	5/4	No real value	$\approx -9.513657 - 9.513657i$
$(-8)^{10/10}$	1	$= -8$	$= -8$
$(-8)^{10/9}$	10/9	≈ 10.079368	$\approx -9.471508 - 3.447347i$

Equivalent fractions give the same value within each convention. Principal complex values agree with real values at integer exponents; noninteger powers with odd reduced denominators select different branches.

Base 8: complete exponent table

Positive real convention. All decimal values marked $\approx$ are rounded to six places. Read down the left block, then down the right block.

TERM	r/s	VALUE
$8^{1/2}$	1/2	≈ 2.828427
$8^{1/3}$	1/3	$= 2$
$8^{1/4}$	1/4	≈ 1.681793
$8^{1/5}$	1/5	≈ 1.515717
$8^{1/6}$	1/6	≈ 1.414214
$8^{1/7}$	1/7	≈ 1.345900
$8^{1/8}$	1/8	≈ 1.296840
$8^{1/9}$	1/9	≈ 1.259921
$8^{1/10}$	1/10	≈ 1.231144
$8^{1/1}$	1	$= 8$
$8^{2/1}$	2	$= 64$
$8^{2/2}$	1	$= 8$
$8^{2/3}$	2/3	$= 4$
$8^{2/4}$	1/2	≈ 2.828427
$8^{2/5}$	2/5	≈ 2.297397
$8^{2/6}$	1/3	$= 2$
$8^{2/7}$	2/7	≈ 1.811447
$8^{2/8}$	1/4	≈ 1.681793
$8^{2/9}$	2/9	≈ 1.587401
$8^{2/10}$	1/5	≈ 1.515717
$8^{3/1}$	3	$= 512$
$8^{3/2}$	3/2	≈ 22.627417
$8^{3/3}$	1	$= 8$
$8^{3/4}$	3/4	≈ 4.756828
$8^{3/5}$	3/5	≈ 3.482202

TERM	r/s	VALUE
$8^{3/6}$	1/2	≈ 2.828427
$8^{3/7}$	3/7	≈ 2.438027
$8^{3/8}$	3/8	≈ 2.181015
$8^{3/9}$	1/3	$= 2$
$8^{3/10}$	3/10	≈ 1.866066
$8^{4/1}$	4	$= 4,096$
$8^{4/2}$	2	$= 64$
$8^{4/3}$	4/3	$= 16$
$8^{4/4}$	1	$= 8$
$8^{4/5}$	4/5	≈ 5.278032
$8^{4/6}$	2/3	$= 4$
$8^{4/7}$	4/7	≈ 3.281341
$8^{4/8}$	1/2	≈ 2.828427
$8^{4/9}$	4/9	≈ 2.519842
$8^{4/10}$	2/5	≈ 2.297397
$8^{5/1}$	5	$= 32,768$
$8^{5/2}$	5/2	≈ 181.019336
$8^{5/3}$	5/3	$= 32$
$8^{5/4}$	5/4	≈ 13.454343
$8^{5/5}$	1	$= 8$
$8^{5/6}$	5/6	≈ 5.656854
$8^{5/7}$	5/7	≈ 4.416358
$8^{5/8}$	5/8	≈ 3.668016
$8^{5/9}$	5/9	≈ 3.174802
$8^{5/10}$	1/2	≈ 2.828427

PATTERN TO NOTICE

Since $8 = 2^3$, reduced denominators 1 and 3 give integers. For example, $8^{2/3} = 4$ and $8^{4/3} = 16$.

Base 8: complete exponent table

Positive real convention. All decimal values marked $\approx$ are rounded to six places. Read down the left block, then down the right block.

TERM	r/s	VALUE
$8^{6/1}$	6	= 262,144
$8^{6/2}$	3	= 512
$8^{6/3}$	2	= 64
$8^{6/4}$	3/2	≈ 22.627417
$8^{6/5}$	6/5	≈ 12.125733
$8^{6/6}$	1	= 8
$8^{6/7}$	6/7	≈ 5.943977
$8^{6/8}$	3/4	≈ 4.756828
$8^{6/9}$	2/3	= 4
$8^{6/10}$	3/5	≈ 3.482202
$8^{7/1}$	7	= 2,097,152
$8^{7/2}$	7/2	$\approx 1,448.154688$
$8^{7/3}$	7/3	= 128
$8^{7/4}$	7/4	≈ 38.054628
$8^{7/5}$	7/5	≈ 18.379174
$8^{7/6}$	7/6	≈ 11.313708
$8^{7/7}$	1	= 8
$8^{7/8}$	7/8	≈ 6.168843
$8^{7/9}$	7/9	≈ 5.039684
$8^{7/10}$	7/10	≈ 4.287094
$8^{8/1}$	8	= 16,777,216
$8^{8/2}$	4	= 4,096
$8^{8/3}$	8/3	= 256
$8^{8/4}$	2	= 64
$8^{8/5}$	8/5	≈ 27.857618

TERM	r/s	VALUE
$8^{8/6}$	4/3	= 16
$8^{8/7}$	8/7	≈ 10.767202
$8^{8/8}$	1	= 8
$8^{8/9}$	8/9	≈ 6.349604
$8^{8/10}$	4/5	≈ 5.278032
$8^{9/1}$	9	= 134,217,728
$8^{9/2}$	9/2	$\approx 11,585.237503$
$8^{9/3}$	3	= 512
$8^{9/4}$	9/4	≈ 107.634741
$8^{9/5}$	9/5	≈ 42.224253
$8^{9/6}$	3/2	≈ 22.627417
$8^{9/7}$	9/7	≈ 14.491579
$8^{9/8}$	9/8	≈ 10.374716
$8^{9/9}$	1	= 8
$8^{9/10}$	9/10	≈ 6.498019
$8^{10/1}$	10	= 1,073,741,824
$8^{10/2}$	5	= 32,768
$8^{10/3}$	10/3	= 1,024
$8^{10/4}$	5/2	≈ 181.019336
$8^{10/5}$	2	= 64
$8^{10/6}$	5/3	= 32
$8^{10/7}$	10/7	≈ 19.504218
$8^{10/8}$	5/4	≈ 13.454343
$8^{10/10}$	1	= 8
$8^{10/9}$	10/9	≈ 10.079368

PATTERN TO NOTICE

Since $8 = 2^3$, reduced denominators 1 and 3 give integers. For example, $8^{2/3} = 4$ and $8^{4/3} = 16$.

Base -9: real & principal complex

Reduce p/q to r/s. Odd s permits a real value; even s does not. The principal complex column uses $\text{Arg}(-9) = \pi$.

TERM	r/s	REAL VALUE	PRINCIPAL COMPLEX VALUE
$(-9)^{1/2}$	1/2	No real value	$= 3i$
$(-9)^{1/3}$	1/3	≈ -2.080084	$\approx 1.040042 + 1.801405i$
$(-9)^{1/4}$	1/4	No real value	$\approx 1.224745 + 1.224745i$
$(-9)^{1/5}$	1/5	≈ -1.551846	$\approx 1.255469 + 0.912152i$
$(-9)^{1/6}$	1/6	No real value	$\approx 1.249025 + 0.721125i$
$(-9)^{1/7}$	1/7	≈ -1.368738	$\approx 1.233190 + 0.593873i$
$(-9)^{1/8}$	1/8	No real value	$\approx 1.215894 + 0.503640i$
$(-9)^{1/9}$	1/9	≈ -1.276518	$\approx 1.199535 + 0.436595i$
$(-9)^{1/10}$	1/10	No real value	$\approx 1.184761 + 0.384952i$
$(-9)^{1/1}$	1	$= -9$	$= -9$
$(-9)^{2/1}$	2	$= 81$	$= 81$
$(-9)^{2/2}$	1	$= -9$	$= -9$
$(-9)^{2/3}$	2/3	≈ 4.326749	$\approx -2.163374 + 3.747074i$
$(-9)^{2/4}$	1/2	No real value	$= 3i$
$(-9)^{2/5}$	2/5	≈ 2.408225	$\approx 0.744182 + 2.290358i$
$(-9)^{2/6}$	1/3	≈ -2.080084	$\approx 1.040042 + 1.801405i$
$(-9)^{2/7}$	2/7	≈ 1.873444	$\approx 1.168073 + 1.464718i$
$(-9)^{2/8}$	1/4	No real value	$\approx 1.224745 + 1.224745i$
$(-9)^{2/9}$	2/9	≈ 1.629498	$\approx 1.248268 + 1.047421i$
$(-9)^{2/10}$	1/5	≈ -1.551846	$\approx 1.255469 + 0.912152i$
$(-9)^{3/1}$	3	$= -729$	$= -729$
$(-9)^{3/2}$	3/2	No real value	$= -27i$
$(-9)^{3/3}$	1	$= -9$	$= -9$
$(-9)^{3/4}$	3/4	No real value	$\approx -3.674235 + 3.674235i$
$(-9)^{3/5}$	3/5	≈ -3.737193	$\approx -1.154856 + 3.554282i$
$(-9)^{3/6}$	1/2	No real value	$= 3i$
$(-9)^{3/7}$	3/7	≈ -2.564254	$\approx 0.570600 + 2.499963i$
$(-9)^{3/8}$	3/8	No real value	$\approx 0.872330 + 2.105990i$
$(-9)^{3/9}$	1/3	≈ -2.080084	$\approx 1.040042 + 1.801405i$
$(-9)^{3/10}$	3/10	No real value	$\approx 1.136296 + 1.563977i$
$(-9)^{4/1}$	4	$= 6,561$	$= 6,561$
$(-9)^{4/2}$	2	$= 81$	$= 81$
$(-9)^{4/3}$	4/3	≈ 18.720754	$\approx -9.360377 - 16.212649i$
$(-9)^{4/4}$	1	$= -9$	$= -9$

Exact definition: $P = 9^{r/s} [\cos(\pi r/s) + i \sin(\pi r/s)]$. A nonreal principal value can coexist with a real odd-root value.

Base -9: real & principal complex

Reduce p/q to r/s. Odd s permits a real value; even s does not. The principal complex column uses $\text{Arg}(-9) = \pi$.

TERM	r/s	REAL VALUE	PRINCIPAL COMPLEX VALUE
$(-9)^{4/5}$	4/5	≈ 5.799546	$\approx -4.691931 + 3.408888i$
$(-9)^{4/6}$	2/3	≈ 4.326749	$\approx -2.163374 + 3.747074i$
$(-9)^{4/7}$	4/7	≈ 3.509792	$\approx -0.781002 + 3.421795i$
$(-9)^{4/8}$	1/2	No real value	$= 3i$
$(-9)^{4/9}$	4/9	≈ 2.655264	$\approx 0.461082 + 2.614925i$
$(-9)^{4/10}$	2/5	≈ 2.408225	$\approx 0.744182 + 2.290358i$
$(-9)^{5/1}$	5	$= -59,049$	$= -59,049$
$(-9)^{5/2}$	5/2	No real value	$= 243i$
$(-9)^{5/3}$	5/3	≈ -38.940738	$\approx 19.470369 - 33.723669i$
$(-9)^{5/4}$	5/4	No real value	$\approx -11.022704 - 11.022704i$
$(-9)^{5/5}$	1	$= -9$	$= -9$
$(-9)^{5/6}$	5/6	No real value	$\approx -5.404216 + 3.120126i$
$(-9)^{5/7}$	5/7	≈ -4.803987	$\approx -2.995237 + 3.755908i$
$(-9)^{5/8}$	5/8	No real value	$\approx -1.510919 + 3.647682i$
$(-9)^{5/9}$	5/9	≈ -3.389493	$\approx -0.588579 + 3.337999i$
$(-9)^{5/10}$	1/2	No real value	$= 3i$
$(-9)^{6/1}$	6	$= 531,441$	$= 531,441$
$(-9)^{6/2}$	3	$= -729$	$= -729$
$(-9)^{6/3}$	2	$= 81$	$= 81$
$(-9)^{6/4}$	3/2	No real value	$= -27i$
$(-9)^{6/5}$	6/5	≈ 13.966610	$\approx -11.299225 - 8.209367i$
$(-9)^{6/6}$	1	$= -9$	$= -9$
$(-9)^{6/7}$	6/7	≈ 6.575400	$\approx -5.924230 + 2.852959i$
$(-9)^{6/8}$	3/4	No real value	$\approx -3.674235 + 3.674235i$
$(-9)^{6/9}$	2/3	≈ 4.326749	$\approx -2.163374 + 3.747074i$
$(-9)^{6/10}$	3/5	≈ -3.737193	$\approx -1.154856 + 3.554282i$
$(-9)^{7/1}$	7	$= -4,782,969$	$= -4,782,969$
$(-9)^{7/2}$	7/2	No real value	$= -2,187i$
$(-9)^{7/3}$	7/3	≈ -168.486790	$\approx 84.243395 + 145.913840i$
$(-9)^{7/4}$	7/4	No real value	$\approx 33.068112 - 33.068112i$
$(-9)^{7/5}$	7/5	≈ -21.674022	$\approx -6.697641 - 20.613220i$
$(-9)^{7/6}$	7/6	No real value	$\approx -11.241223 - 6.490123i$
$(-9)^{7/7}$	1	$= -9$	$= -9$

The real column uses $(-1)^r \times 9^{r/s}$ when s is odd. Even reduced numerators give positive real values; odd ones give negative real values.

Base -9: real & principal complex

Reduce p/q to r/s. Odd s permits a real value; even s does not. The principal complex column uses $\text{Arg}(-9) = \pi$.

TERM	r/s	REAL VALUE	PRINCIPAL COMPLEX VALUE
$(-9)^{7/8}$	7/8	No real value	$\approx -6.317970 + 2.616989i$
$(-9)^{7/9}$	7/9	≈ -5.523173	$\approx -4.230996 + 3.550227i$
$(-9)^{7/10}$	7/10	No real value	$\approx -2.736456 + 3.766408i$
$(-9)^{8/1}$	8	$= 43,046,721$	$= 43,046,721$
$(-9)^{8/2}$	4	$= 6,561$	$= 6,561$
$(-9)^{8/3}$	8/3	≈ 350.466646	$\approx -175.233323 + 303.513018i$
$(-9)^{8/4}$	2	$= 81$	$= 81$
$(-9)^{8/5}$	8/5	≈ 33.634735	$\approx 10.393705 - 31.988534i$
$(-9)^{8/6}$	4/3	≈ 18.720754	$\approx -9.360377 - 16.212649i$
$(-9)^{8/7}$	8/7	≈ 12.318643	$\approx -11.098714 - 5.344859i$
$(-9)^{8/8}$	1	$= -9$	$= -9$
$(-9)^{8/9}$	8/9	≈ 7.050429	$\approx -6.625236 + 2.411389i$
$(-9)^{8/10}$	4/5	≈ 5.799546	$\approx -4.691931 + 3.408888i$
$(-9)^{9/1}$	9	$= -387,420,489$	$= -387,420,489$
$(-9)^{9/2}$	9/2	No real value	$= 19,683i$
$(-9)^{9/3}$	3	$= -729$	$= -729$
$(-9)^{9/4}$	9/4	No real value	$\approx 99.204335 + 99.204335i$
$(-9)^{9/5}$	9/5	≈ -52.195915	$\approx 42.227382 - 30.679989i$
$(-9)^{9/6}$	3/2	No real value	$= -27i$
$(-9)^{9/7}$	9/7	≈ -16.860996	$\approx -10.512659 - 13.182458i$
$(-9)^{9/8}$	9/8	No real value	$\approx -10.943045 - 4.532757i$
$(-9)^{9/9}$	1	$= -9$	$= -9$
$(-9)^{9/10}$	9/10	No real value	$\approx -6.871073 + 2.232547i$
$(-9)^{10/1}$	10	$= 3,486,784,401$	$= 3,486,784,401$
$(-9)^{10/2}$	5	$= -59,049$	$= -59,049$
$(-9)^{10/3}$	10/3	$\approx 1,516.381107$	$\approx -758.190554 - 1,313.224560i$
$(-9)^{10/4}$	5/2	No real value	$= 243i$
$(-9)^{10/5}$	2	$= 81$	$= 81$
$(-9)^{10/6}$	5/3	≈ -38.940738	$\approx 19.470369 - 33.723669i$
$(-9)^{10/7}$	10/7	≈ 23.078288	$\approx -5.135402 - 22.499667i$
$(-9)^{10/8}$	5/4	No real value	$\approx -11.022704 - 11.022704i$
$(-9)^{10/10}$	1	$= -9$	$= -9$
$(-9)^{10/9}$	10/9	≈ 11.488662	$\approx -10.795811 - 3.929354i$

Equivalent fractions give the same value within each convention. Principal complex values agree with real values at integer exponents; noninteger powers with odd reduced denominators select different branches.

Base 9: complete exponent table

Positive real convention. All decimal values marked $\approx$ are rounded to six places. Read down the left block, then down the right block.

TERM	r/s	VALUE
$9^{1/2}$	1/2	= 3
$9^{1/3}$	1/3	≈ 2.080084
$9^{1/4}$	1/4	≈ 1.732051
$9^{1/5}$	1/5	≈ 1.551846
$9^{1/6}$	1/6	≈ 1.442250
$9^{1/7}$	1/7	≈ 1.368738
$9^{1/8}$	1/8	≈ 1.316074
$9^{1/9}$	1/9	≈ 1.276518
$9^{1/10}$	1/10	≈ 1.245731
$9^{1/1}$	1	= 9
$9^{2/1}$	2	= 81
$9^{2/2}$	1	= 9
$9^{2/3}$	2/3	≈ 4.326749
$9^{2/4}$	1/2	= 3
$9^{2/5}$	2/5	≈ 2.408225
$9^{2/6}$	1/3	≈ 2.080084
$9^{2/7}$	2/7	≈ 1.873444
$9^{2/8}$	1/4	≈ 1.732051
$9^{2/9}$	2/9	≈ 1.629498
$9^{2/10}$	1/5	≈ 1.551846
$9^{3/1}$	3	= 729
$9^{3/2}$	3/2	= 27
$9^{3/3}$	1	= 9
$9^{3/4}$	3/4	≈ 5.196152
$9^{3/5}$	3/5	≈ 3.737193

TERM	r/s	VALUE
$9^{3/6}$	1/2	= 3
$9^{3/7}$	3/7	≈ 2.564254
$9^{3/8}$	3/8	≈ 2.279507
$9^{3/9}$	1/3	≈ 2.080084
$9^{3/10}$	3/10	≈ 1.933182
$9^{4/1}$	4	= 6,561
$9^{4/2}$	2	= 81
$9^{4/3}$	4/3	≈ 18.720754
$9^{4/4}$	1	= 9
$9^{4/5}$	4/5	≈ 5.799546
$9^{4/6}$	2/3	≈ 4.326749
$9^{4/7}$	4/7	≈ 3.509792
$9^{4/8}$	1/2	= 3
$9^{4/9}$	4/9	≈ 2.655264
$9^{4/10}$	2/5	≈ 2.408225
$9^{5/1}$	5	= 59,049
$9^{5/2}$	5/2	= 243
$9^{5/3}$	5/3	≈ 38.940738
$9^{5/4}$	5/4	≈ 15.588457
$9^{5/5}$	1	= 9
$9^{5/6}$	5/6	≈ 6.240251
$9^{5/7}$	5/7	≈ 4.803987
$9^{5/8}$	5/8	≈ 3.948222
$9^{5/9}$	5/9	≈ 3.389493
$9^{5/10}$	1/2	= 3

PATTERN TO NOTICE

Since $9 = 3^2$, reduced denominators 1 and 2 give integers. For example, $9^{1/2} = 3$ and $9^{3/2} = 27$.

Base 9: complete exponent table

Positive real convention. All decimal values marked $\approx$ are rounded to six places. Read down the left block, then down the right block.

TERM	r/s	VALUE
$9^{6/1}$	6	= 531,441
$9^{6/2}$	3	= 729
$9^{6/3}$	2	= 81
$9^{6/4}$	$3/2$	= 27
$9^{6/5}$	$6/5$	≈ 13.966610
$9^{6/6}$	1	= 9
$9^{6/7}$	$6/7$	≈ 6.575400
$9^{6/8}$	$3/4$	≈ 5.196152
$9^{6/9}$	$2/3$	≈ 4.326749
$9^{6/10}$	$3/5$	≈ 3.737193
$9^{7/1}$	7	= 4,782,969
$9^{7/2}$	$7/2$	= 2,187
$9^{7/3}$	$7/3$	≈ 168.486790
$9^{7/4}$	$7/4$	≈ 46.765372
$9^{7/5}$	$7/5$	≈ 21.674022
$9^{7/6}$	$7/6$	≈ 12.980246
$9^{7/7}$	1	= 9
$9^{7/8}$	$7/8$	≈ 6.838521
$9^{7/9}$	$7/9$	≈ 5.523173
$9^{7/10}$	$7/10$	≈ 4.655537
$9^{8/1}$	8	= 43,046,721
$9^{8/2}$	4	= 6,561
$9^{8/3}$	$8/3$	≈ 350.466646
$9^{8/4}$	2	= 81
$9^{8/5}$	$8/5$	≈ 33.634735

TERM	r/s	VALUE
$9^{8/6}$	$4/3$	≈ 18.720754
$9^{8/7}$	$8/7$	≈ 12.318643
$9^{8/8}$	1	= 9
$9^{8/9}$	$8/9$	≈ 7.050429
$9^{8/10}$	$4/5$	≈ 5.799546
$9^{9/1}$	9	= 387,420,489
$9^{9/2}$	$9/2$	= 19,683
$9^{9/3}$	3	= 729
$9^{9/4}$	$9/4$	≈ 140.296115
$9^{9/5}$	$9/5$	≈ 52.195915
$9^{9/6}$	$3/2$	= 27
$9^{9/7}$	$9/7$	≈ 16.860996
$9^{9/8}$	$9/8$	≈ 11.844666
$9^{9/9}$	1	= 9
$9^{9/10}$	$9/10$	≈ 7.224674
$9^{10/1}$	10	= 3,486,784,401
$9^{10/2}$	5	= 59,049
$9^{10/3}$	$10/3$	$\approx 1,516.381107$
$9^{10/4}$	$5/2$	= 243
$9^{10/5}$	2	= 81
$9^{10/6}$	$5/3$	≈ 38.940738
$9^{10/7}$	$10/7$	≈ 23.078288
$9^{10/8}$	$5/4$	≈ 15.588457
$9^{10/10}$	1	= 9
$9^{10/9}$	$10/9$	≈ 11.488662

PATTERN TO NOTICE

Since $9 = 3^2$, reduced denominators 1 and 2 give integers. For example, $9^{1/2} = 3$ and $9^{3/2} = 27$.

Base -10: real & principal complex

Reduce p/q to r/s. Odd s permits a real value; even s does not. The principal complex column uses $\text{Arg}(-10) = \pi$.

TERM	r/s	REAL VALUE	PRINCIPAL COMPLEX VALUE
$(-10)^{1/2}$	1/2	No real value	$\approx 3.162278i$
$(-10)^{1/3}$	1/3	≈ -2.154435	$\approx 1.077217 + 1.865795i$
$(-10)^{1/4}$	1/4	No real value	$\approx 1.257433 + 1.257433i$
$(-10)^{1/5}$	1/5	≈ -1.584893	$\approx 1.282206 + 0.931577i$
$(-10)^{1/6}$	1/6	No real value	$\approx 1.271151 + 0.733900i$
$(-10)^{1/7}$	1/7	≈ -1.389495	$\approx 1.251892 + 0.602880i$
$(-10)^{1/8}$	1/8	No real value	$\approx 1.232013 + 0.510317i$
$(-10)^{1/9}$	1/9	≈ -1.291550	$\approx 1.213660 + 0.441736i$
$(-10)^{1/10}$	1/10	No real value	$\approx 1.197309 + 0.389029i$
$(-10)^{1/1}$	1	$= -10$	$= -10$
$(-10)^{2/1}$	2	$= 100$	$= 100$
$(-10)^{2/2}$	1	$= -10$	$= -10$
$(-10)^{2/3}$	2/3	≈ 4.641589	$\approx -2.320794 + 4.019734i$
$(-10)^{2/4}$	1/2	No real value	$\approx 3.162278i$
$(-10)^{2/5}$	2/5	≈ 2.511886	$\approx 0.776216 + 2.388946i$
$(-10)^{2/6}$	1/3	≈ -2.154435	$\approx 1.077217 + 1.865795i$
$(-10)^{2/7}$	2/7	≈ 1.930698	$\approx 1.203770 + 1.509480i$
$(-10)^{2/8}$	1/4	No real value	$\approx 1.257433 + 1.257433i$
$(-10)^{2/9}$	2/9	≈ 1.668101	$\approx 1.277839 + 1.072234i$
$(-10)^{2/10}$	1/5	≈ -1.584893	$\approx 1.282206 + 0.931577i$
$(-10)^{3/1}$	3	$= -1,000$	$= -1,000$
$(-10)^{3/2}$	3/2	No real value	$\approx -31.622777i$
$(-10)^{3/3}$	1	$= -10$	$= -10$
$(-10)^{3/4}$	3/4	No real value	$\approx -3.976354 + 3.976354i$
$(-10)^{3/5}$	3/5	≈ -3.981072	$\approx -1.230219 + 3.786224i$
$(-10)^{3/6}$	1/2	No real value	$\approx 3.162278i$
$(-10)^{3/7}$	3/7	≈ -2.682696	$\approx 0.596956 + 2.615435i$
$(-10)^{3/8}$	3/8	No real value	$\approx 0.907485 + 2.190864i$
$(-10)^{3/9}$	1/3	≈ -2.154435	$\approx 1.077217 + 1.865795i$
$(-10)^{3/10}$	3/10	No real value	$\approx 1.172786 + 1.614201i$
$(-10)^{4/1}$	4	$= 10,000$	$= 10,000$
$(-10)^{4/2}$	2	$= 100$	$= 100$
$(-10)^{4/3}$	4/3	≈ 21.544347	$\approx -10.772173 - 18.657952i$
$(-10)^{4/4}$	1	$= -10$	$= -10$

Exact definition: $P = 10^{r/s} [\cos(\pi r/s) + i \sin(\pi r/s)]$. A nonreal principal value can coexist with a real odd-root value.

Base -10: real & principal complex

Reduce p/q to r/s. Odd s permits a real value; even s does not. The principal complex column uses $\text{Arg}(-10) = \pi$.

TERM	r/s	REAL VALUE	PRINCIPAL COMPLEX VALUE
$(-10)^{4/5}$	4/5	≈ 6.309573	$\approx -5.104552 + 3.708674i$
$(-10)^{4/6}$	2/3	≈ 4.641589	$\approx -2.320794 + 4.019734i$
$(-10)^{4/7}$	4/7	≈ 3.727594	$\approx -0.829468 + 3.634135i$
$(-10)^{4/8}$	1/2	No real value	$\approx 3.162278i$
$(-10)^{4/9}$	4/9	≈ 2.782559	$\approx 0.483186 + 2.740286i$
$(-10)^{4/10}$	2/5	≈ 2.511886	$\approx 0.776216 + 2.388946i$
$(-10)^{5/1}$	5	$= -100,000$	$= -100,000$
$(-10)^{5/2}$	5/2	No real value	$\approx 316.227766i$
$(-10)^{5/3}$	5/3	≈ -46.415888	$\approx 23.207944 - 40.197338i$
$(-10)^{5/4}$	5/4	No real value	$\approx -12.574334 - 12.574334i$
$(-10)^{5/5}$	1	$= -10$	$= -10$
$(-10)^{5/6}$	5/6	No real value	$\approx -5.900162 + 3.406460i$
$(-10)^{5/7}$	5/7	≈ -5.179475	$\approx -3.229350 + 4.049476i$
$(-10)^{5/8}$	5/8	No real value	$\approx -1.613763 + 3.895968i$
$(-10)^{5/9}$	5/9	≈ -3.593814	$\approx -0.624059 + 3.539216i$
$(-10)^{5/10}$	1/2	No real value	$\approx 3.162278i$
$(-10)^{6/1}$	6	$= 1,000,000$	$= 1,000,000$
$(-10)^{6/2}$	3	$= -1,000$	$= -1,000$
$(-10)^{6/3}$	2	$= 100$	$= 100$
$(-10)^{6/4}$	3/2	No real value	$\approx -31.622777i$
$(-10)^{6/5}$	6/5	≈ 15.848932	$\approx -12.822055 - 9.315768i$
$(-10)^{6/6}$	1	$= -10$	$= -10$
$(-10)^{6/7}$	6/7	≈ 7.196857	$\approx -6.484144 + 3.122599i$
$(-10)^{6/8}$	3/4	No real value	$\approx -3.976354 + 3.976354i$
$(-10)^{6/9}$	2/3	≈ 4.641589	$\approx -2.320794 + 4.019734i$
$(-10)^{6/10}$	3/5	≈ -3.981072	$\approx -1.230219 + 3.786224i$
$(-10)^{7/1}$	7	$= -10,000,000$	$= -10,000,000$
$(-10)^{7/2}$	7/2	No real value	$\approx -3,162.277660i$
$(-10)^{7/3}$	7/3	≈ -215.443469	$\approx 107.721735 + 186.579517i$
$(-10)^{7/4}$	7/4	No real value	$\approx 39.763536 - 39.763536i$
$(-10)^{7/5}$	7/5	≈ -25.118864	$\approx -7.762156 - 23.889460i$
$(-10)^{7/6}$	7/6	No real value	$\approx -12.711515 - 7.338996i$
$(-10)^{7/7}$	1	$= -10$	$= -10$

The real column uses $(-1)^r \times 10^{r/s}$ when s is odd. Even reduced numerators give positive real values; odd ones give negative real values.

Base -10: real & principal complex

Reduce p/q to r/s. Odd s permits a real value; even s does not. The principal complex column uses $\text{Arg}(-10) = \pi$.

TERM	r/s	REAL VALUE	PRINCIPAL COMPLEX VALUE
$(-10)^{7/8}$	7/8	No real value	$\approx -6.928119 + 2.869721i$
$(-10)^{7/9}$	7/9	≈ -5.994843	$\approx -4.592316 + 3.853410i$
$(-10)^{7/10}$	7/10	No real value	$\approx -2.945905 + 4.054690i$
$(-10)^{8/1}$	8	$= 100,000,000$	$= 100,000,000$
$(-10)^{8/2}$	4	$= 10,000$	$= 10,000$
$(-10)^{8/3}$	8/3	≈ 464.158883	$\approx -232.079442 + 401.973384i$
$(-10)^{8/4}$	2	$= 100$	$= 100$
$(-10)^{8/5}$	8/5	≈ 39.810717	$\approx 12.302188 - 37.862242i$
$(-10)^{8/6}$	4/3	≈ 21.544347	$\approx -10.772173 - 18.657952i$
$(-10)^{8/7}$	8/7	≈ 13.894955	$\approx -12.518922 - 6.028795i$
$(-10)^{8/8}$	1	$= -10$	$= -10$
$(-10)^{8/9}$	8/9	≈ 7.742637	$\approx -7.275699 + 2.648138i$
$(-10)^{8/10}$	4/5	≈ 6.309573	$\approx -5.104552 + 3.708674i$
$(-10)^{9/1}$	9	$= -1,000,000,000$	$= -1,000,000,000$
$(-10)^{9/2}$	9/2	No real value	$\approx 31,622.776602i$
$(-10)^{9/3}$	3	$= -1,000$	$= -1,000$
$(-10)^{9/4}$	9/4	No real value	$\approx 125.743343 + 125.743343i$
$(-10)^{9/5}$	9/5	≈ -63.095734	$\approx 51.045521 - 37.086742i$
$(-10)^{9/6}$	3/2	No real value	$\approx -31.622777i$
$(-10)^{9/7}$	9/7	≈ -19.306977	$\approx -12.037703 - 15.094803i$
$(-10)^{9/8}$	9/8	No real value	$\approx -12.320132 - 5.103166i$
$(-10)^{9/9}$	1	$= -10$	$= -10$
$(-10)^{9/10}$	9/10	No real value	$\approx -7.554510 + 2.454609i$
$(-10)^{10/1}$	10	$= 10,000,000,000$	$= 10,000,000,000$
$(-10)^{10/2}$	5	$= -100,000$	$= -100,000$
$(-10)^{10/3}$	10/3	$\approx 2,154.434690$	$\approx -1,077.217345 - 1,865.795172i$
$(-10)^{10/4}$	5/2	No real value	$\approx 316.227766i$
$(-10)^{10/5}$	2	$= 100$	$= 100$
$(-10)^{10/6}$	5/3	≈ -46.415888	$\approx 23.207944 - 40.197338i$
$(-10)^{10/7}$	10/7	≈ 26.826958	$\approx -5.969560 - 26.154350i$
$(-10)^{10/8}$	5/4	No real value	$\approx -12.574334 - 12.574334i$
$(-10)^{10/10}$	1	$= -10$	$= -10$
$(-10)^{10/9}$	10/9	≈ 12.915497	$\approx -12.136597 - 4.417360i$

Equivalent fractions give the same value within each convention. Principal complex values agree with real values at integer exponents; noninteger powers with odd reduced denominators select different branches.

Base 10: complete exponent table

Positive real convention. All decimal values marked $\approx$ are rounded to six places. Read down the left block, then down the right block.

TERM	r/s	VALUE
$10^{1/2}$	1/2	≈ 3.162278
$10^{1/3}$	1/3	≈ 2.154435
$10^{1/4}$	1/4	≈ 1.778279
$10^{1/5}$	1/5	≈ 1.584893
$10^{1/6}$	1/6	≈ 1.467799
$10^{1/7}$	1/7	≈ 1.389495
$10^{1/8}$	1/8	≈ 1.333521
$10^{1/9}$	1/9	≈ 1.291550
$10^{1/10}$	1/10	≈ 1.258925
$10^{1/1}$	1	$= 10$
$10^{2/1}$	2	$= 100$
$10^{2/2}$	1	$= 10$
$10^{2/3}$	2/3	≈ 4.641589
$10^{2/4}$	1/2	≈ 3.162278
$10^{2/5}$	2/5	≈ 2.511886
$10^{2/6}$	1/3	≈ 2.154435
$10^{2/7}$	2/7	≈ 1.930698
$10^{2/8}$	1/4	≈ 1.778279
$10^{2/9}$	2/9	≈ 1.668101
$10^{2/10}$	1/5	≈ 1.584893
$10^{3/1}$	3	$= 1,000$
$10^{3/2}$	3/2	≈ 31.622777
$10^{3/3}$	1	$= 10$
$10^{3/4}$	3/4	≈ 5.623413
$10^{3/5}$	3/5	≈ 3.981072

TERM	r/s	VALUE
$10^{3/6}$	1/2	≈ 3.162278
$10^{3/7}$	3/7	≈ 2.682696
$10^{3/8}$	3/8	≈ 2.371374
$10^{3/9}$	1/3	≈ 2.154435
$10^{3/10}$	3/10	≈ 1.995262
$10^{4/1}$	4	$= 10,000$
$10^{4/2}$	2	$= 100$
$10^{4/3}$	4/3	≈ 21.544347
$10^{4/4}$	1	$= 10$
$10^{4/5}$	4/5	≈ 6.309573
$10^{4/6}$	2/3	≈ 4.641589
$10^{4/7}$	4/7	≈ 3.727594
$10^{4/8}$	1/2	≈ 3.162278
$10^{4/9}$	4/9	≈ 2.782559
$10^{4/10}$	2/5	≈ 2.511886
$10^{5/1}$	5	$= 100,000$
$10^{5/2}$	5/2	≈ 316.227766
$10^{5/3}$	5/3	≈ 46.415888
$10^{5/4}$	5/4	≈ 17.782794
$10^{5/5}$	1	$= 10$
$10^{5/6}$	5/6	≈ 6.812921
$10^{5/7}$	5/7	≈ 5.179475
$10^{5/8}$	5/8	≈ 4.216965
$10^{5/9}$	5/9	≈ 3.593814
$10^{5/10}$	1/2	≈ 3.162278

PATTERN TO NOTICE

Since $10 = 2 \times 5$, noninteger reduced exponents give irrational values. Denominator 1 gives familiar integer powers of ten.

Base 10: complete exponent table

Positive real convention. All decimal values marked $\approx$ are rounded to six places. Read down the left block, then down the right block.

TERM	r/s	VALUE	TERM	r/s	VALUE
$10^{6/1}$	6	= 1,000,000	$10^{8/6}$	4/3	≈ 21.544347
$10^{6/2}$	3	= 1,000	$10^{8/7}$	8/7	≈ 13.894955
$10^{6/3}$	2	= 100	$10^{8/8}$	1	= 10
$10^{6/4}$	3/2	≈ 31.622777	$10^{8/9}$	8/9	≈ 7.742637
$10^{6/5}$	6/5	≈ 15.848932	$10^{8/10}$	4/5	≈ 6.309573
$10^{6/6}$	1	= 10	$10^{9/1}$	9	= 1,000,000,000
$10^{6/7}$	6/7	≈ 7.196857	$10^{9/2}$	9/2	$\approx 31,622.776602$
$10^{6/8}$	3/4	≈ 5.623413	$10^{9/3}$	3	= 1,000
$10^{6/9}$	2/3	≈ 4.641589	$10^{9/4}$	9/4	≈ 177.827941
$10^{6/10}$	3/5	≈ 3.981072	$10^{9/5}$	9/5	≈ 63.095734
$10^{7/1}$	7	= 10,000,000	$10^{9/6}$	3/2	≈ 31.622777
$10^{7/2}$	7/2	$\approx 3,162.277660$	$10^{9/7}$	9/7	≈ 19.306977
$10^{7/3}$	7/3	≈ 215.443469	$10^{9/8}$	9/8	≈ 13.335214
$10^{7/4}$	7/4	≈ 56.234133	$10^{9/9}$	1	= 10
$10^{7/5}$	7/5	≈ 25.118864	$10^{9/10}$	9/10	≈ 7.943282
$10^{7/6}$	7/6	≈ 14.677993	$10^{10/1}$	10	= 10,000,000,000
$10^{7/7}$	1	= 10	$10^{10/2}$	5	= 100,000
$10^{7/8}$	7/8	≈ 7.498942	$10^{10/3}$	10/3	$\approx 2,154.434690$
$10^{7/9}$	7/9	≈ 5.994843	$10^{10/4}$	5/2	≈ 316.227766
$10^{7/10}$	7/10	≈ 5.011872	$10^{10/5}$	2	= 100
$10^{8/1}$	8	= 100,000,000	$10^{10/6}$	5/3	≈ 46.415888
$10^{8/2}$	4	= 10,000	$10^{10/7}$	10/7	≈ 26.826958
$10^{8/3}$	8/3	≈ 464.158883	$10^{10/8}$	5/4	≈ 17.782794
$10^{8/4}$	2	= 100	$10^{10/10}$	1	= 10
$10^{8/5}$	8/5	≈ 39.810717	$10^{10/9}$	10/9	≈ 12.915497

Closing calculation: the exponent is one plus one ninth. The corresponding value is approximately 12.915497.

$$10^{10/9}$$