

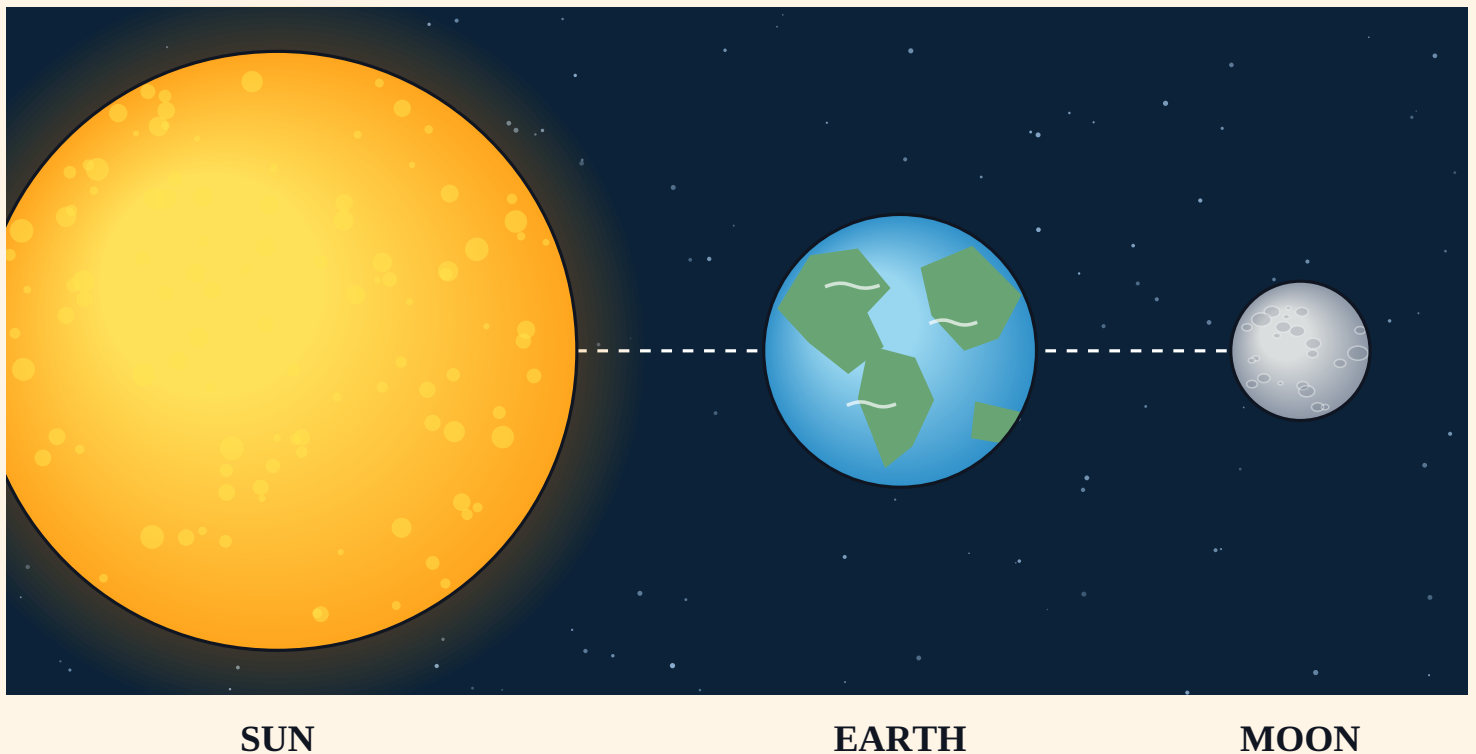
One Formula, Three Worlds

Gravity Connects the Sun, Earth & Moon

$$F = \frac{GMm}{r^2}$$

MASS × MASS

SQUARE OF CENTER DISTANCE



One interaction. Three connected worlds.



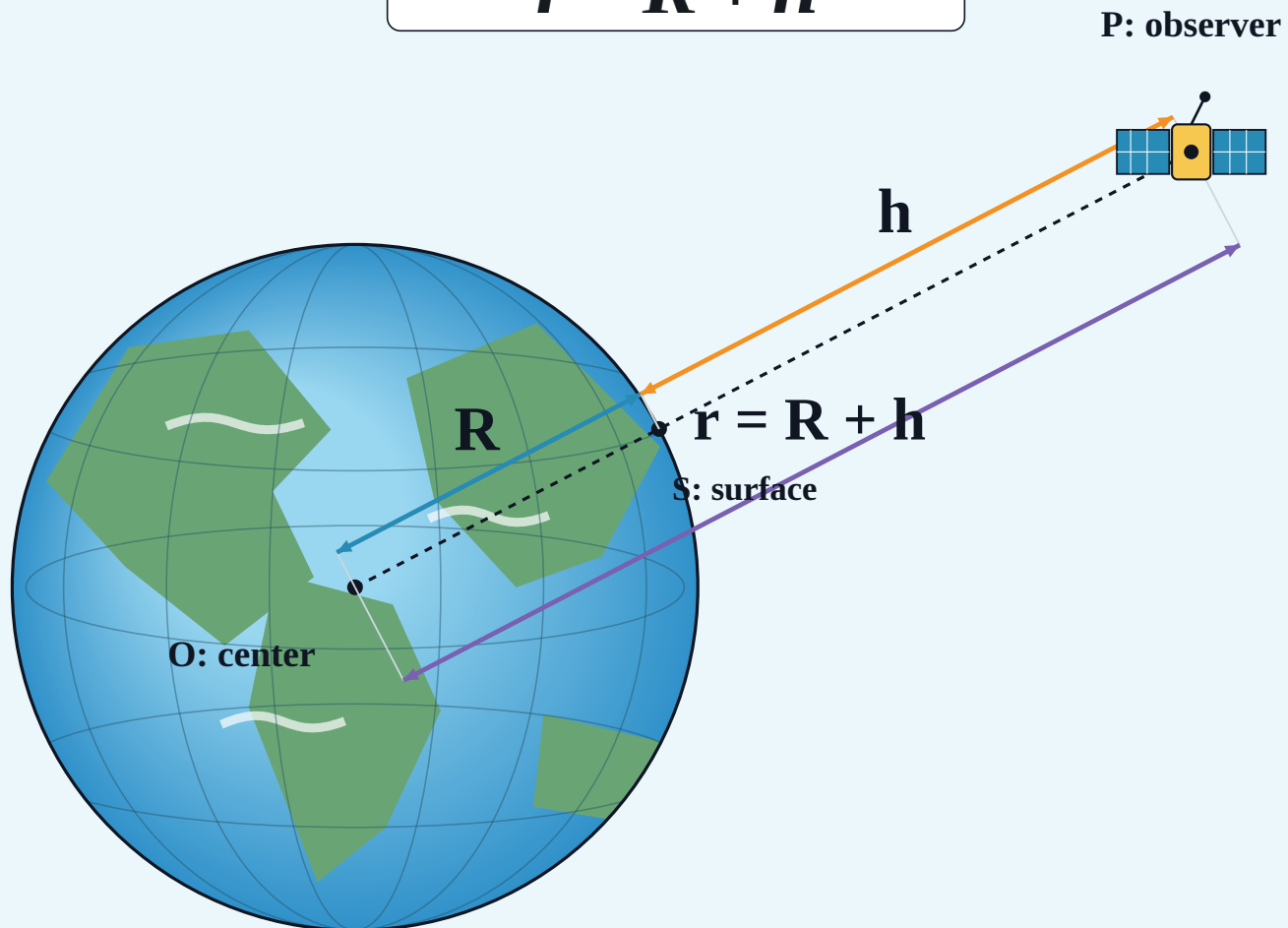
Newtonian gravity describes mutual attraction between masses. The distance r runs between their centers. This artistic arrangement is not to scale; each illustrated force pair acts on two different bodies.

Source: OpenStax, University Physics 1, 13.1

The Body's Size Is Not Your Location

Radius R • Height h • Center Distance r

$$r = R + h$$



R: body radius

h: radial height

r: center distance

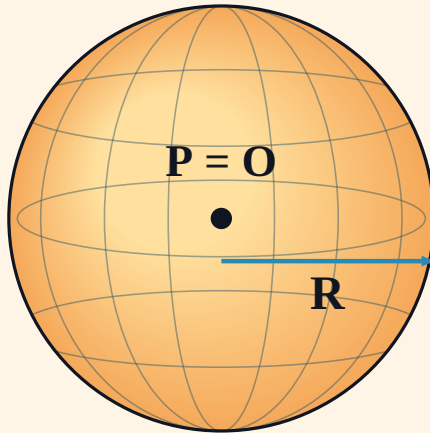
$$F = \frac{GMm}{(R + h)^2}$$

R reaches the surface; **h** continues outward to the observation point; **r** covers the whole trip. For a spherical body in ordinary spatial geometry, $r = R + h$. Schematic; the satellite is enlarged.

Source: OpenStax, University Physics 1, 13.2

One Body, Four Possible Locations

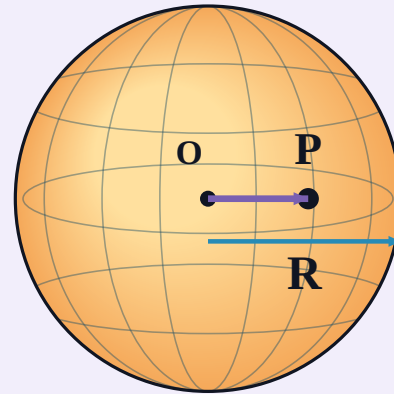
At the Center



$$r = 0$$

The point P and center O coincide. Here $r = 0$; this location is not the body's surface.

Inside the Body

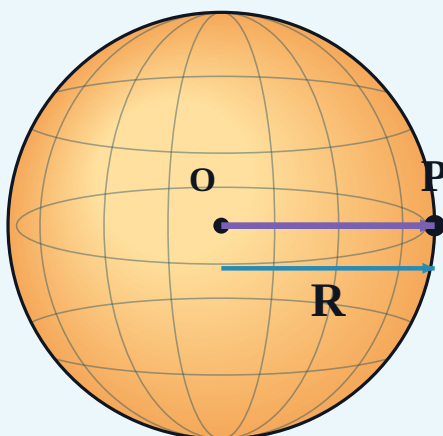


$$0 < r < R$$

The point lies before the surface: $r < R$. An interior gravity calculation needs the enclosed mass, not automatically all of M .

Source: OpenStax, University Physics 1, 13.2

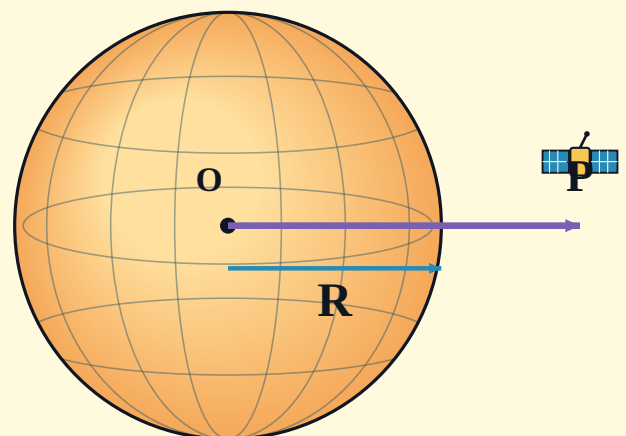
On the Surface



$$r = R$$

The point reaches the edge: $r = R$ and $h = 0$. "Need not equal" does not mean "can never equal."

Beyond the Surface

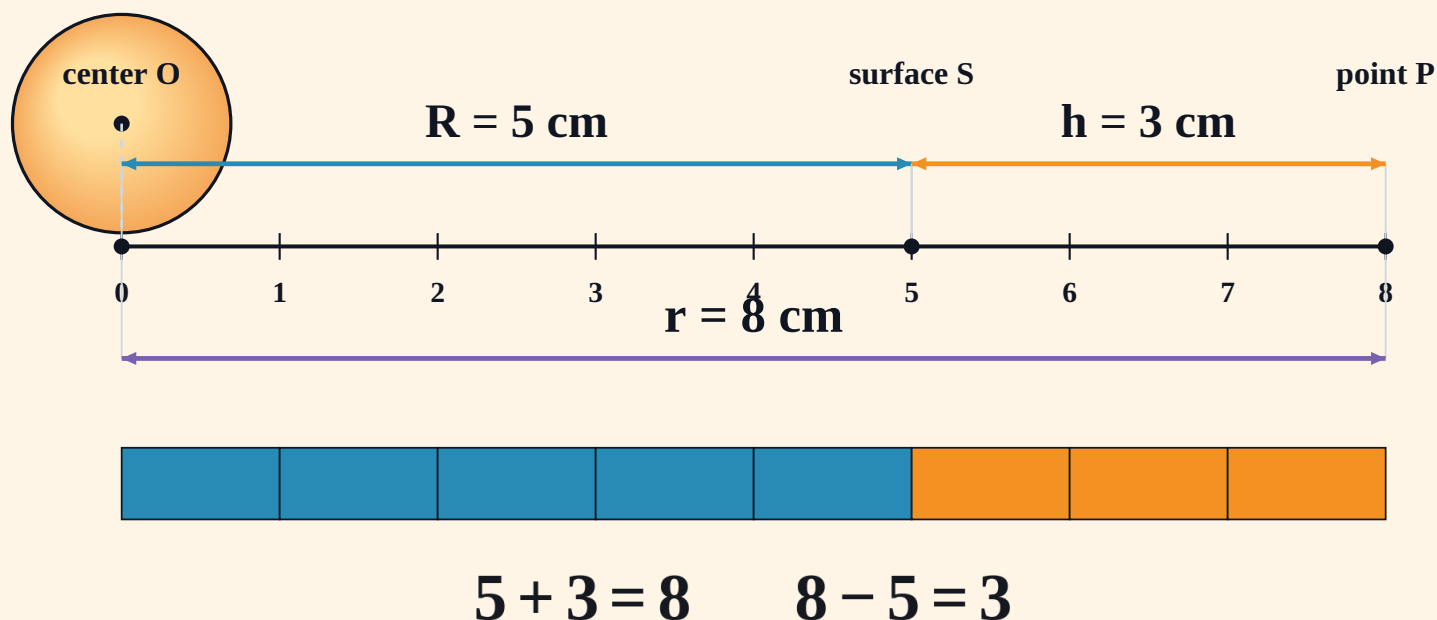


$$r > R$$

Moving the observer outward increases r . The body itself does not grow, so its radius R stays unchanged. Schematic.

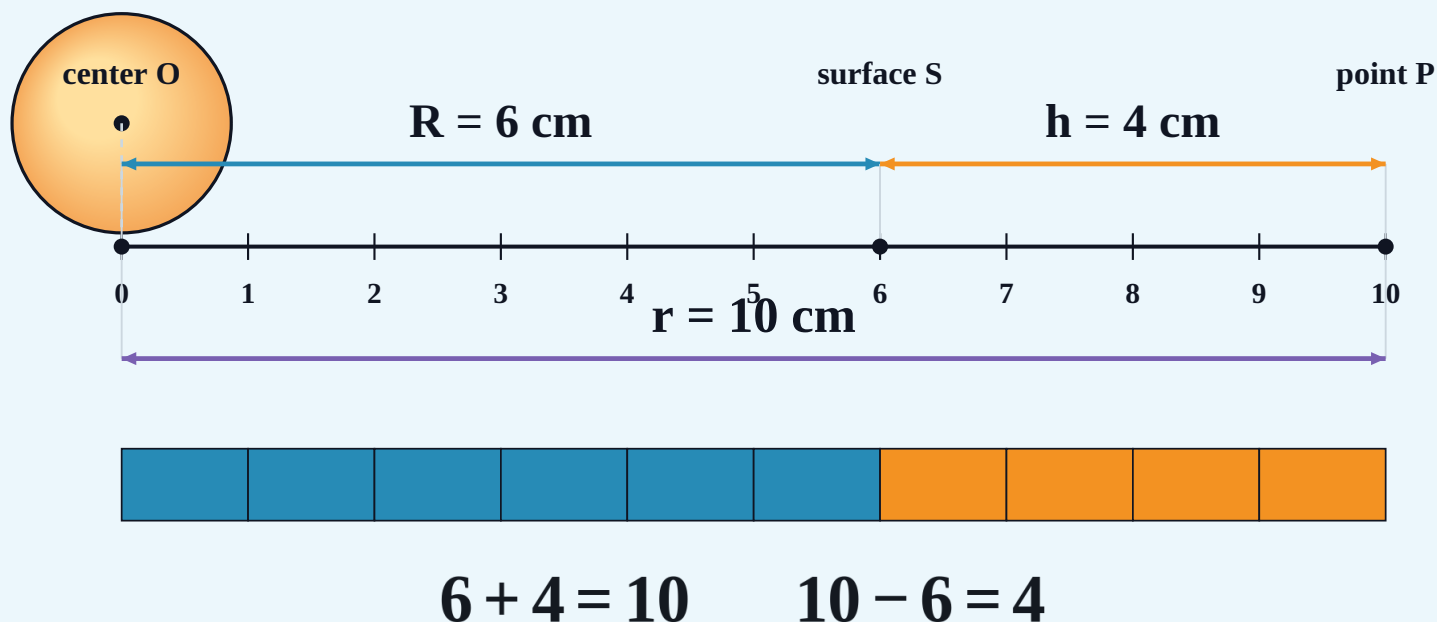
See Addition & Subtraction as Distances

Two Consecutive Lengths Make One Trip



A radius of 5 cm followed by a height of 3 cm makes a center distance of 8 cm. The numbered radial line is drawn to scale; the body icon is not.

Subtract the Body's Radius to Find Height

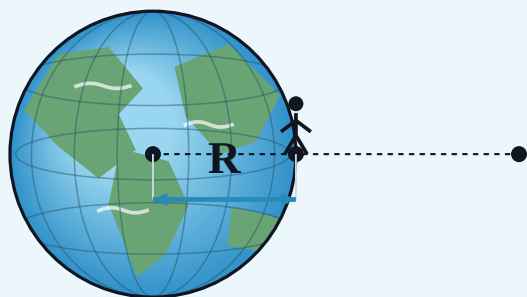


The point is 10 cm from the center, and the surface is 6 cm from the center. Their difference is the 4 cm height. The numbered radial line is drawn to scale.

Same Classroom Earth, Different Heights

At Ground Level

$$h = 0 \text{ km}$$



$$r = 6,400 \text{ km}$$

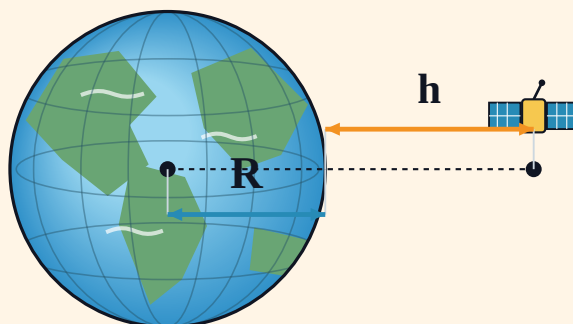
$$g/g_R = 1$$

Classroom model throughout this page: $R = 6,400 \text{ km}$ and surface gravity $g_R = 9.80 \text{ m/s}^2$. At $h = 0$, r is R —not zero. Schematics.

Source: OpenStax, University Physics 1, 13.2

400 km Above the Surface

$$h = 400 \text{ km}$$



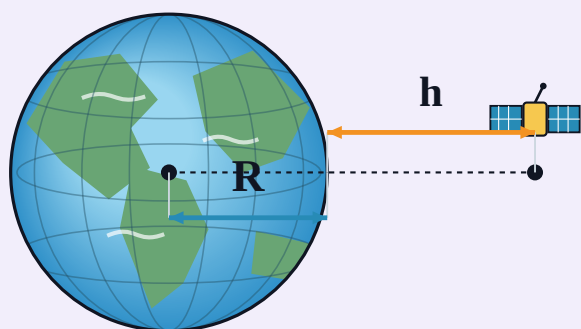
$$r = 6,800 \text{ km}$$

$$g/g_R = 256/289$$

The correct center distance is 6,800 km, not 400 km. Gravity is $(6,400/6,800)^2 \approx 88.58\%$ of its surface value.

One Radius Above the Surface

$$h = 6,400 \text{ km}$$



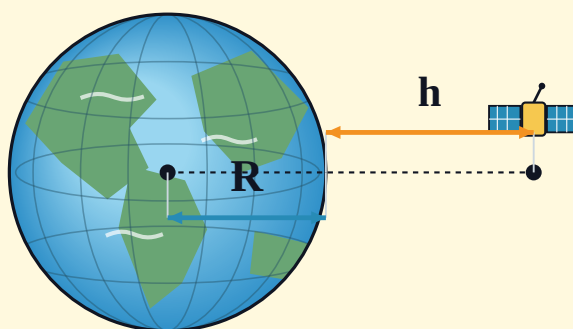
$$r = 12,800 \text{ km}$$

$$g/g_R = 1/4$$

The height equals R , but the center distance equals $2R$. The gravitational field is one fourth of its surface value.

Three Radii Above the Surface

$$h = 19,200 \text{ km}$$



$$r = 25,600 \text{ km}$$

$$g/g_R = 1/16$$

Adding the body's radius gives $r = 4R$. The inverse square leaves $1/16$, or 6.25%, of the surface field.

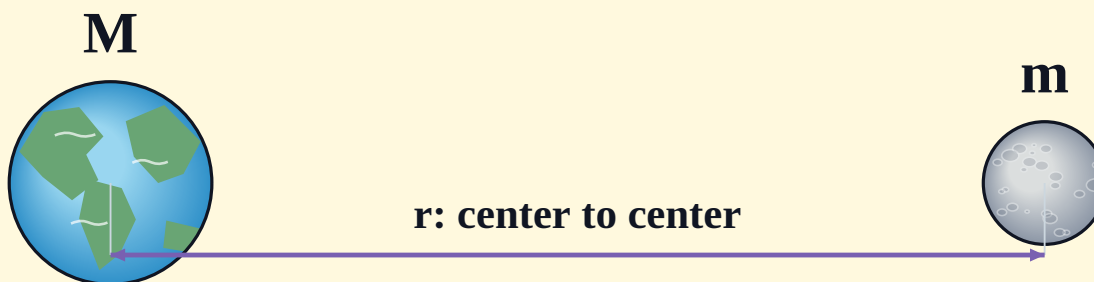
Read the Gravitational Fraction

Multiply Above • Square Below • Divide

NUMERATOR: multiply the constant and masses

$$F = \frac{G \times M \times m}{r^2}$$

DENOMINATOR: $r \times r$, the distance squared



$$K = GMm \quad F(r) = \frac{K}{r^2}$$

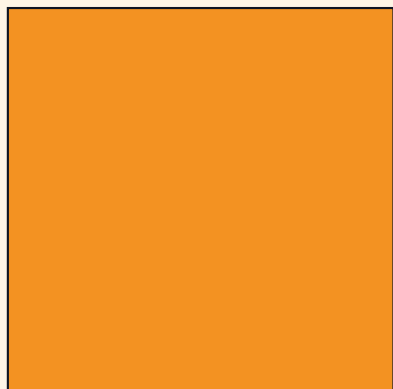
For fixed $K = GMm$, K/r^2 is a rational function of r : a quotient of polynomials. That does not make every output a rational number. This exterior spherical model requires $r \geq R > 0$.

Source: OpenStax, University Physics 1, 13.1

Make the Inverse Square Visible

One Center Distance

$$r = R$$

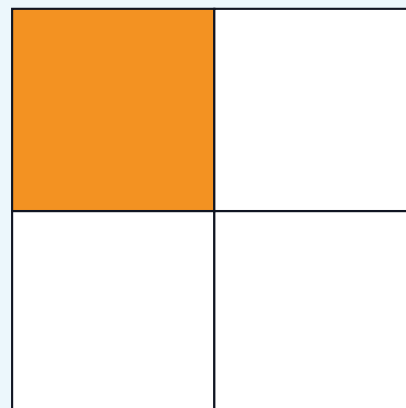


$$g/g_R = 1$$

The entire reference field is 1. These grids represent fractions, not pieces of a material gravitational substance. Keep the body and masses fixed.

Twice the Center Distance

$$r = 2R$$

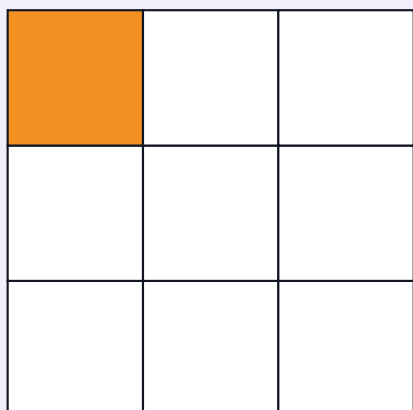


$$\frac{g}{g_R} = \frac{1}{2^2} = \frac{1}{4}$$

Two squares along each edge make four equal squares. One shaded square represents 1/4 of the reference field.

Three Times the Center Distance

$$r = 3R$$

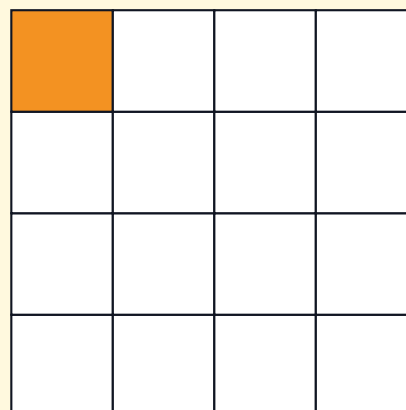


$$\frac{g}{g_R} = \frac{1}{3^2} = \frac{1}{9}$$

Three rows of three make nine equal squares. The field fraction is 1/9, approximately 11.11%.

Four Times the Center Distance

$$r = 4R$$

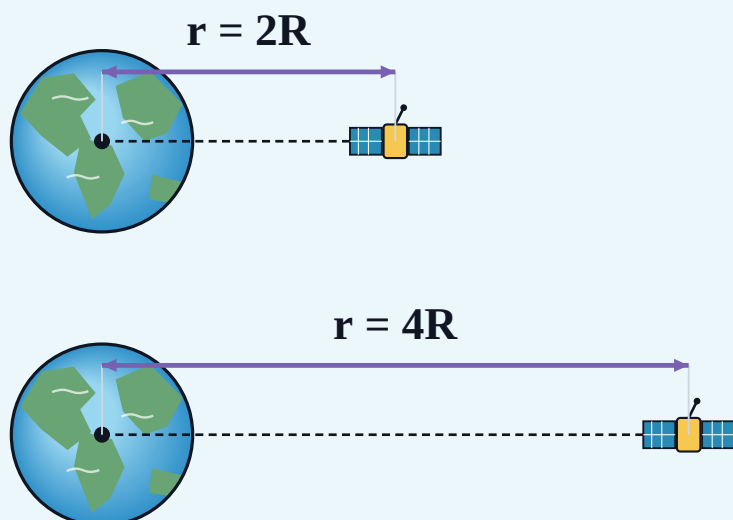


$$\frac{g}{g_R} = \frac{1}{4^2} = \frac{1}{16}$$

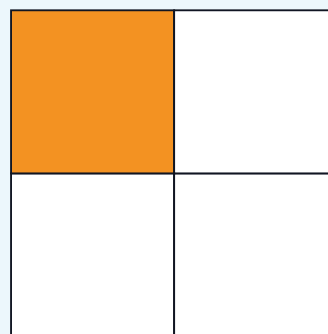
Four rows of four make sixteen equal squares. The field fraction is 1/16, exactly 6.25%.

Double Which Distance?

Doubling r Gives One Fourth the Force



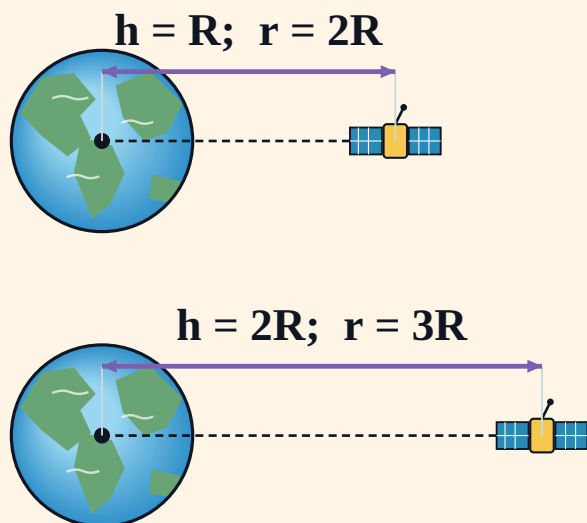
$$F_2/F_1 = (2/4)^2 = 1/4$$



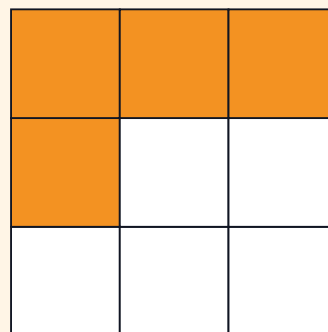
Fraction of the earlier force

The observer moves from center distance $2R$ to $4R$. With the masses fixed, the force ratio is $(2R/4R)^2 = 1/4$. In each comparison, the distance scale is consistent; body icons are schematic.

Doubling h Usually Gives a Different Fraction



$$F_2/F_1 = (2/3)^2 = 4/9$$



Fraction of the earlier force

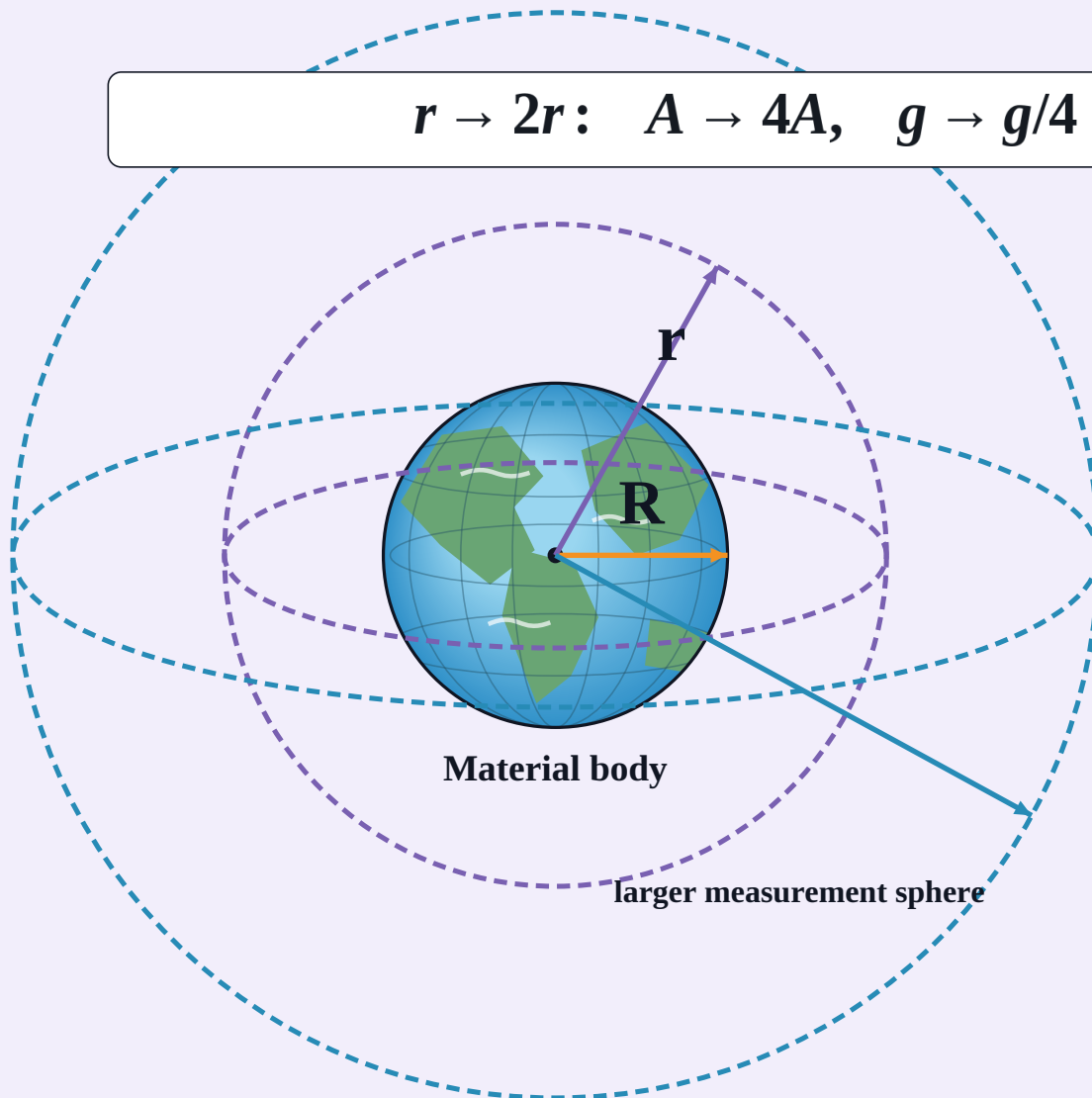
Height changes from R to $2R$, but center distance changes from $2R$ to $3R$. Therefore the new force is $4/9$ of the old force—not $1/4$.

A Measurement Sphere Is Not More Planet

The Solid Body Uses R ; the Surrounding Surface Uses r

$$A = 4\pi r^2$$

$$r \rightarrow 2r: A \rightarrow 4A, g \rightarrow g/4$$



$$gA = 4\pi GM$$

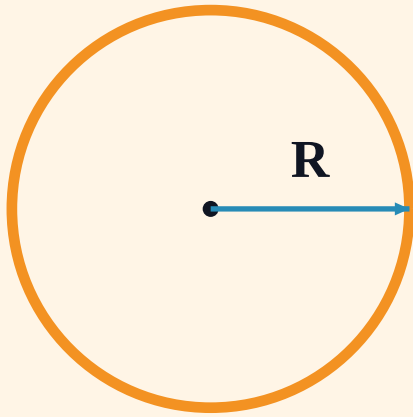
$$F = \frac{GMm}{r^2} = \frac{4\pi GMm}{A}$$

The dashed spheres mark observation locations, not new material. For a spherical exterior field, area grows as r^2 while field strength falls as $1/r^2$. Rewriting the law with sphere area requires the factor 4π . Schematic.

Source: OpenStax, University Physics 1, 13.1

Length, Area & Volume Are Different

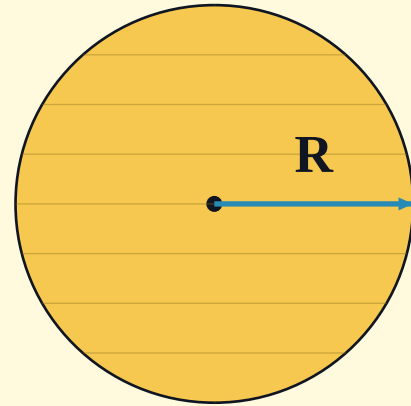
Around the Edge



$$C = 2\pi R$$

Circumference measures a length around a circle. Its units are meters, centimeters, or another length unit.

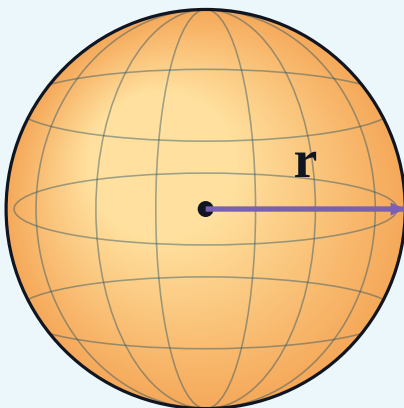
Across the Flat Disk



$$A_{\text{disk}} = \pi R^2$$

Disk area measures a two-dimensional region. Doubling R multiplies its area by four; square units are required.

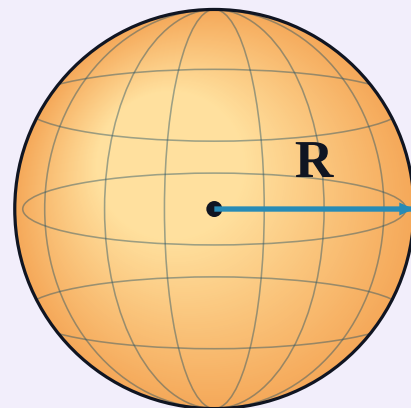
Over the Spherical Surface



$$A_{\text{sphere}} = 4\pi r^2$$

The surrounding observation sphere has radius r and area $4\pi r^2$. Its surface area is not the body's volume.

Inside the Spherical Body



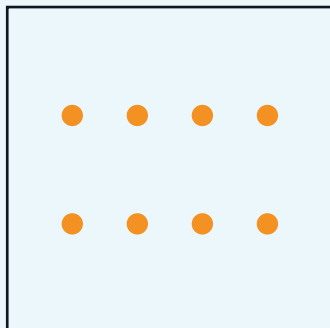
$$V = \frac{4}{3}\pi R^3$$

The material body has radius R and volume $(4/3)\pi R^3$. Doubling R multiplies its volume by eight.

Density Connects Mass to Size

Same Volume, Different Amounts of Mass

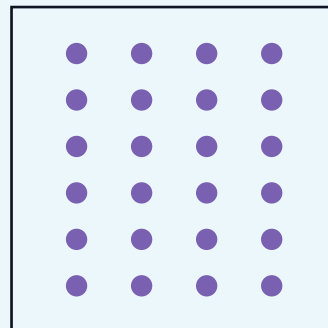
Same volume V



Mass M

$$\bar{\rho} = \frac{M}{V}$$

Same volume V



Mass $3M$

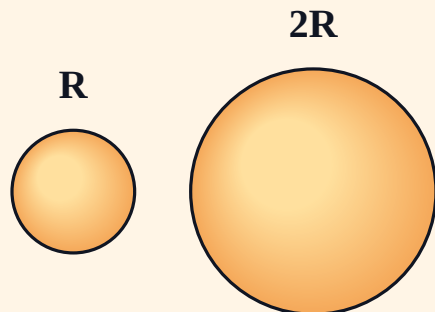
$$\bar{\rho} \rightarrow 3\bar{\rho}$$

Equal-sized boxes can contain different masses. Average density is total mass divided by total volume. Dots stand for equal mass portions; they are not a literal microscopic model.

Source: OpenStax, University Physics 1, 14.1

Changing R Requires a Stated Condition

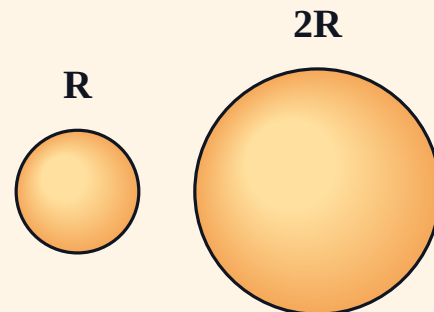
FIXED DENSITY



$$M \rightarrow 8M$$

$$g_R \rightarrow 2g_R$$

FIXED TOTAL MASS



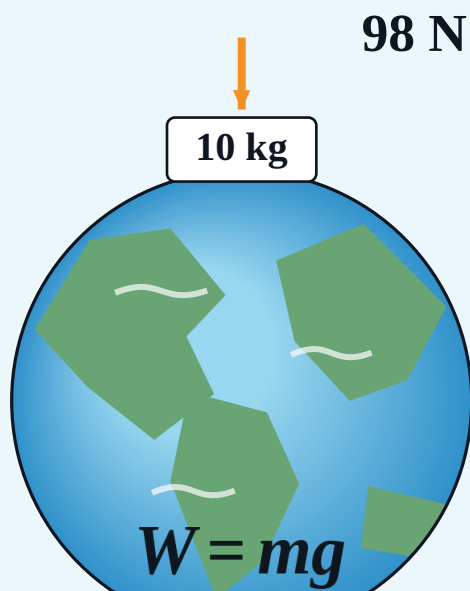
$$M \rightarrow M$$

$$g_R \rightarrow g_R/4$$

For spherical bodies, doubling R gives eight times the mass at fixed average density, so surface gravity doubles. At fixed total mass, the same radius change makes surface gravity one fourth as large.

Mass Stays; Weight Can Change

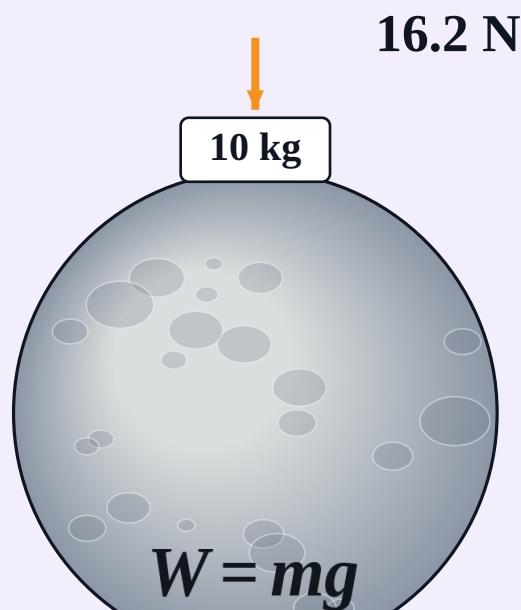
The Package on Earth



At $g \approx 9.80 \text{ m/s}^2$, a 10 kg package has gravitational weight 98 N. Kilograms describe mass; newtons describe force. Schematic.

Source: OpenStax, University Physics 1, 13.2

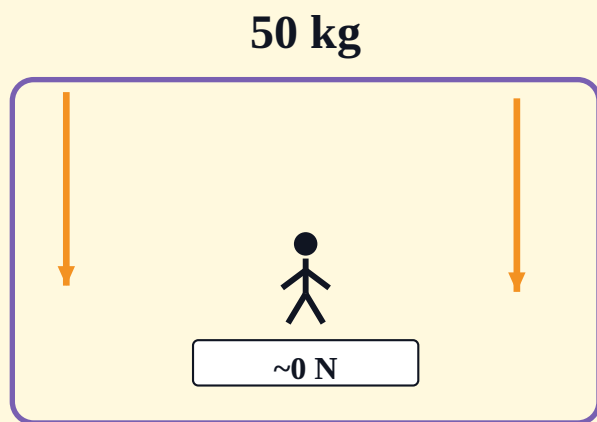
The Same Package on the Moon



At lunar $g \approx 1.62 \text{ m/s}^2$, that same 10 kg package weighs about 16.2 N. Its mass has not become one sixth as large.

Source: NASA Science, Moon Facts

Floating Does Not Erase Gravity

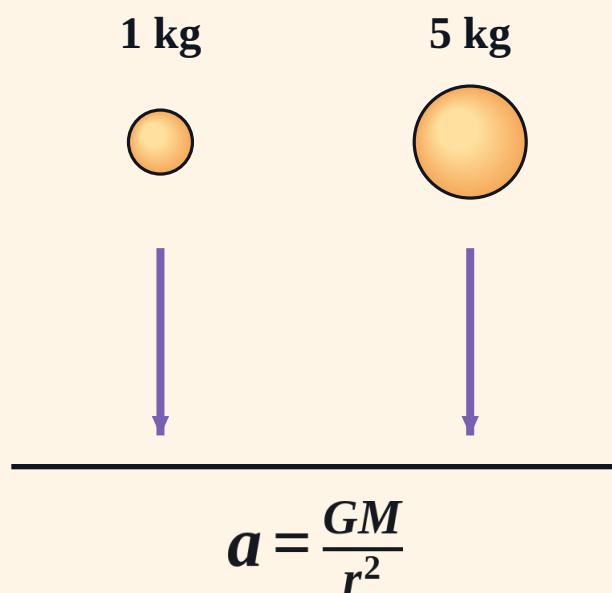


$$F_g \approx 434 \text{ N}$$

In the classroom Earth model, a 50 kg object at 400 km has gravitational force about 434 N. A freely falling support scale can still read nearly zero.

Source: OpenStax, University Physics 1, 13.4

Same Free-Fall Acceleration

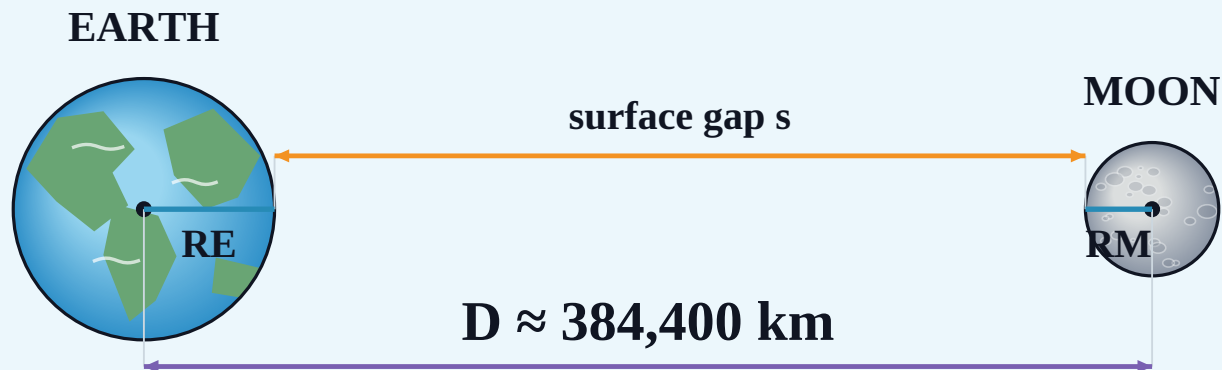


Neglecting air resistance, tiny test objects at the same location have the same gravitational acceleration. Their forces differ in proportion to their masses.

Source: OpenStax, University Physics 1, 13.2

Use the Centers of Both Bodies

Earth–Moon: Center Distance Is Not the Empty Gap

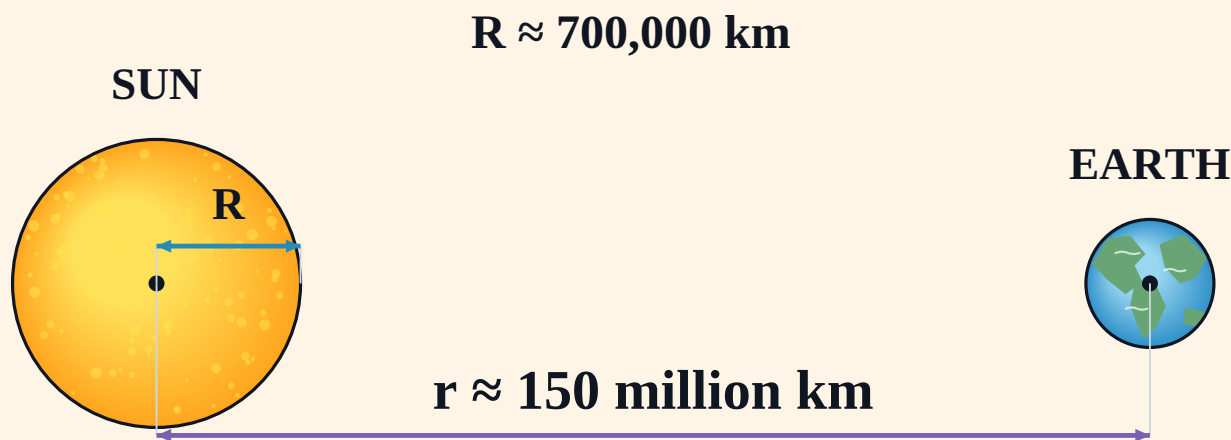


$$D = R_E + s + R_M$$

A representative mean center separation is 384,400 km. With $R_E = 6,371 \text{ km}$ and $R_M \approx 1,740 \text{ km}$, the facing-surface gap is about 376,289 km. Use D , not that gap, in the force law. Not to scale.

Source: NASA/JPL, Planetary Physical Parameters | NASA Science, Moon Facts

Sun–Earth: Solar Radius Is Not Orbital Distance



$$r \neq R$$

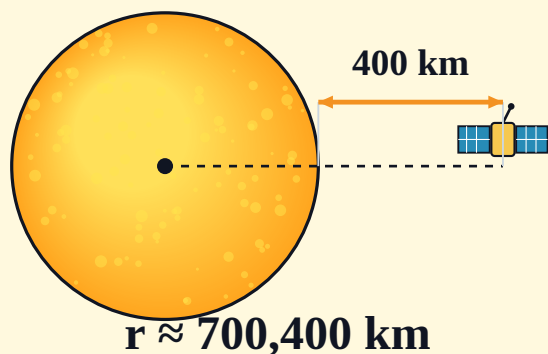
The Sun's reference radius is about 700,000 km; Earth is about 150 million km from its center. The solar reference surface is the photosphere, not solid ground. Sizes and separation are not to scale.

Source: NASA Science, Sun Facts

Equal Heights, Unequal Gravity Fractions

Above the Sun's Reference Surface

$$R \approx 700,000 \text{ km}$$



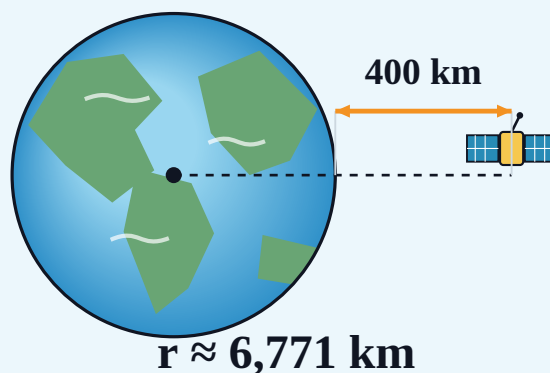
99.89% of surface gravity

Rounded spherical model: 400 km is tiny compared with the Sun's radius. The fraction shown is relative to the Sun's own reference-surface gravity; atmospheric mass is neglected.

Source: NASA Science, Sun Facts

400 km Above Earth

$$R \approx 6,371 \text{ km}$$



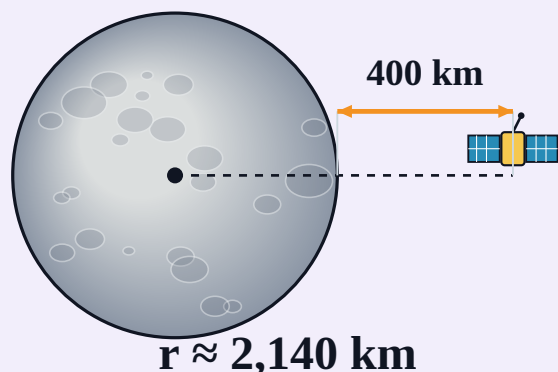
88.53% of surface gravity

Using the mean radius 6,371 km gives $r = 6,771 \text{ km}$. This reference-radius example differs slightly from the deliberately rounded classroom Earth.

Source: NASA/JPL, Planetary Physical Parameters

400 km Above the Moon

$$R \approx 1,740 \text{ km}$$



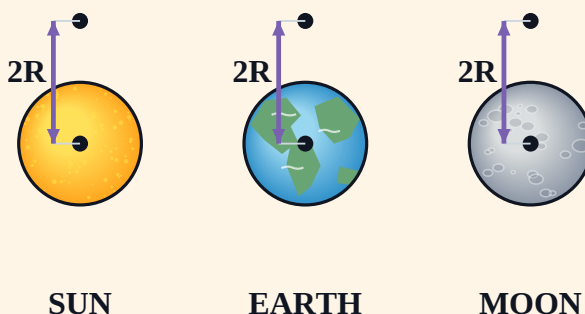
66.11% of surface gravity

Using the rounded lunar radius 1,740 km gives $r = 2,140 \text{ km}$. The same altitude occupies a larger fraction of this smaller body's radius.

Source: NASA Science, Moon Facts

At Height $h = R$

$$h = R \Rightarrow r = 2R$$

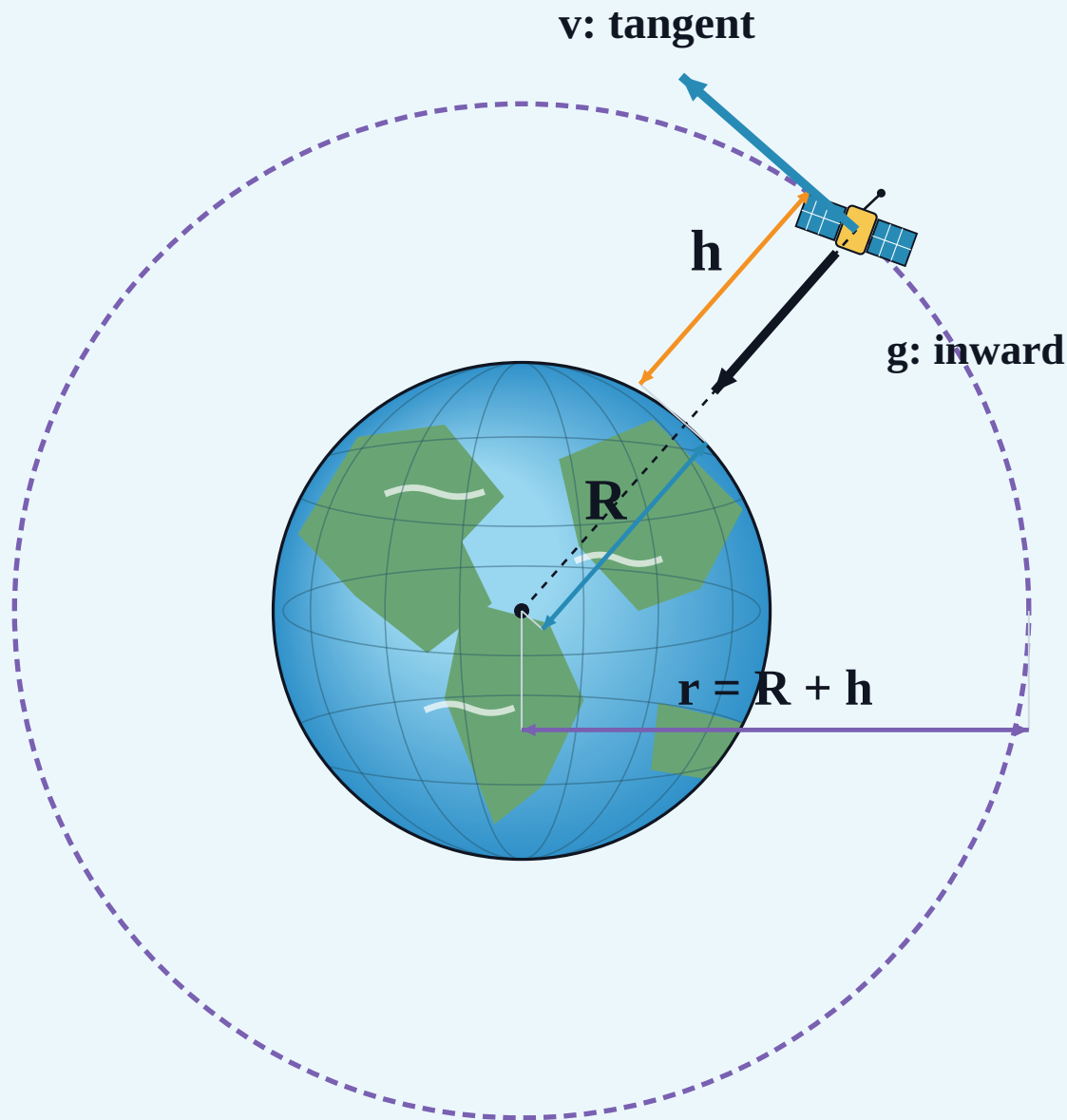


$$g/g_R = 1/4$$

For any of these idealized spherical exterior fields, $h = R$ gives $r = 2R$ and $g/g_R = 1/4$. Equal fractions do not imply equal accelerations. Schematics.

An Orbit Is a Continuing Free Fall

Velocity Goes Along the Path; Gravity Points Inward



$$\frac{GMm}{r^2} = \frac{mv^2}{r}$$

$$v = \sqrt{\frac{GM}{r}}$$

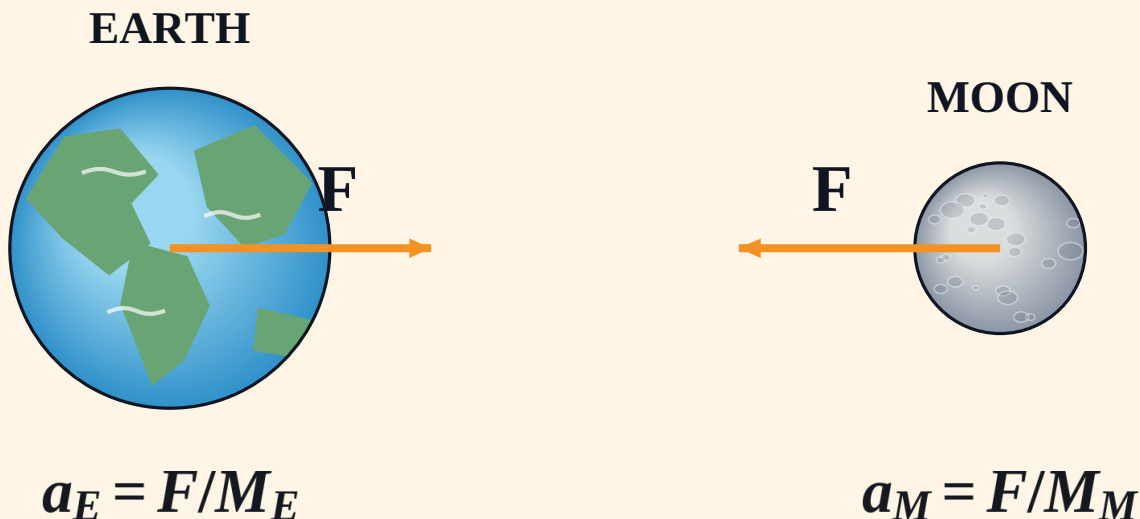
For a small satellite in a circular orbit around a much larger spherical body, gravity supplies the centripetal force. “Centripetal” names the inward net force; it is not an extra competing force. Drag and other perturbations are neglected. Not to scale.

Source: OpenStax, University Physics 1, 13.4

A Force Has Both Size & Direction

Equal Attraction, Different Accelerations

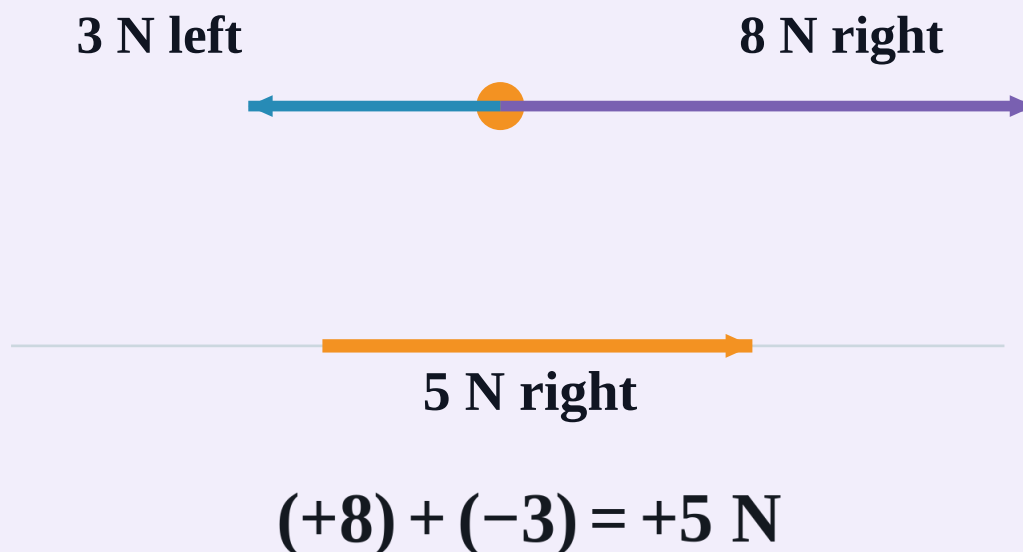
Equal forces act on different bodies.



Earth and Moon pull on each other with equal force magnitudes in opposite directions. Because their masses differ, $a = F/m$ gives different accelerations. Equal-length force arrows act on different bodies; sizes are schematic.

Source: OpenStax, University Physics 1, 13.1

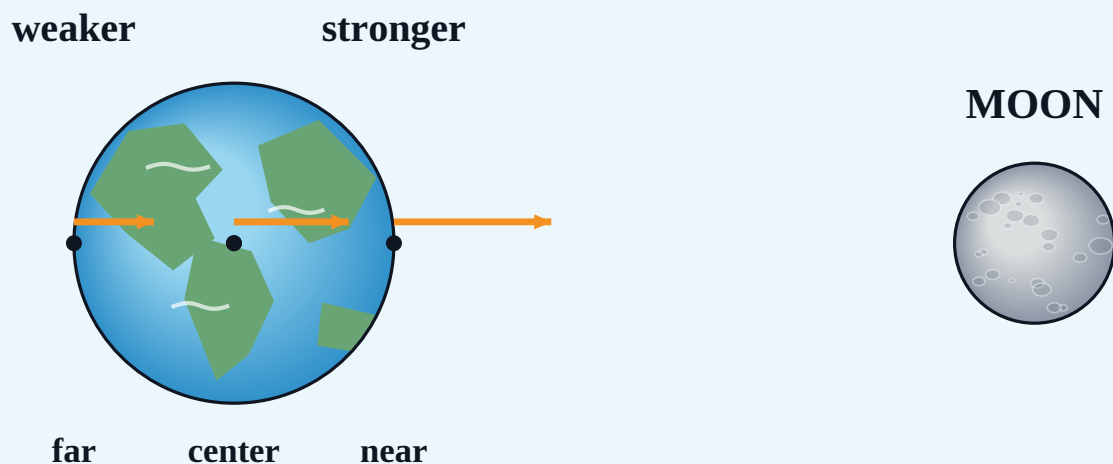
Combine Opposing Arrows with Signed Addition



Taking rightward as positive, an 8 N rightward force and a 3 N leftward force give a 5 N rightward net force. Adding magnitudes alone would discard direction.

Tides Compare Gravity from Place to Place

First Compare the Moon's Actual Pull



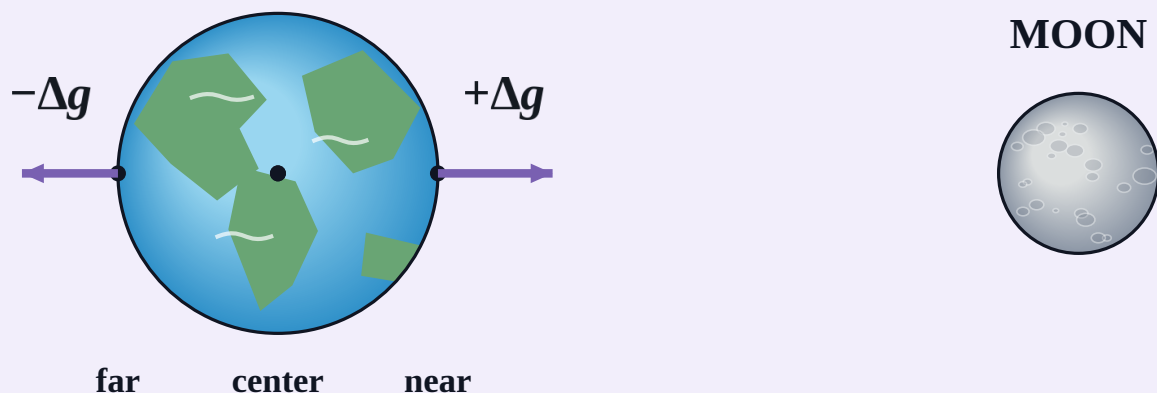
$$g_{\text{near}} > g_{\text{center}} > g_{\text{far}}$$

The Moon attracts the near side of Earth more strongly than the center and the far side. Every orange arrow points toward the Moon. Earth's own gravitational field is not drawn. Schematics; differences are exaggerated.

Source: OpenStax, University Physics 1, 13.6

Then Subtract the Center's Acceleration

Subtract the center's acceleration



$$\Delta g \approx \frac{2GM\ell}{r^3} \quad (\ell \ll r)$$

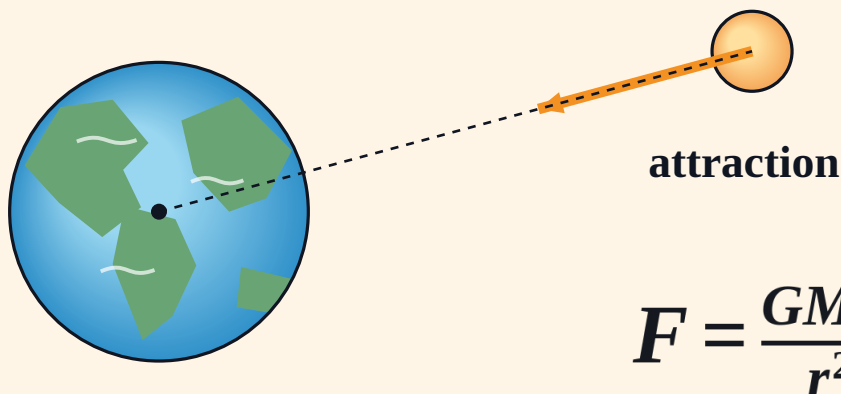
Relative to Earth's center, the near and far sides stretch apart along the Earth–Moon line. The formula estimates a small radial field difference across length ℓ . Real ocean tides also depend on water depth, coastlines, and motion.

Source: OpenStax, University Physics 1, 13.6

Newton & Einstein: Two Descriptions

Newtonian Gravity: A Force Model

Force on a mass

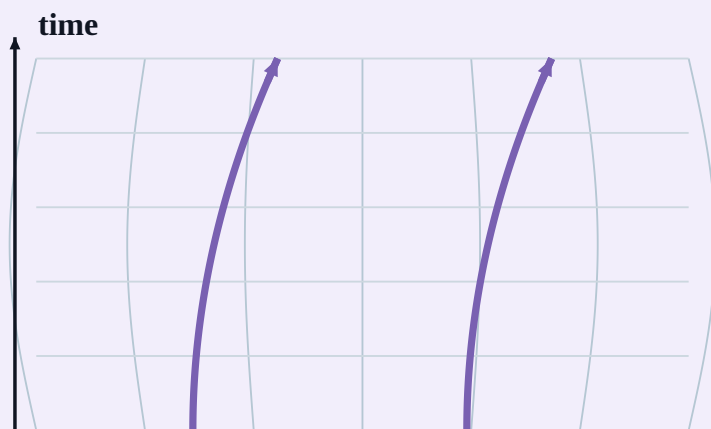


Weak fields • Low speeds

Newton's model represents gravity as attraction between masses. It is an excellent approximation for many weak-field, low-speed situations. The positive scalar formula gives the force magnitude; the arrow supplies direction. Schematic.

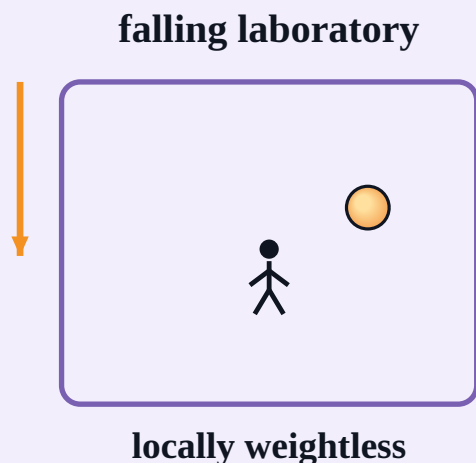
Source: OpenStax, University Physics 1, 13.1

General Relativity: Geometry & Free Fall



Free-fall paths in spacetime

Curved spacetime • Local free fall

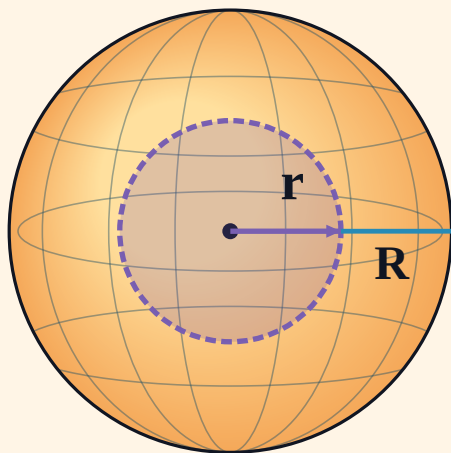


General relativity links matter and energy to spacetime geometry. Freely falling objects follow geodesics. A sufficiently small falling laboratory is locally weightless; tidal effects can remain. The schematic spacetime grid is not a material sheet.

Source: Einstein Online, Weightlessness to Curvature

Inside the Body, Change the Mass Model

An Interior Point Uses Enclosed Mass



$$M_{\text{enc}} = M \left(\frac{r}{R} \right)^3$$

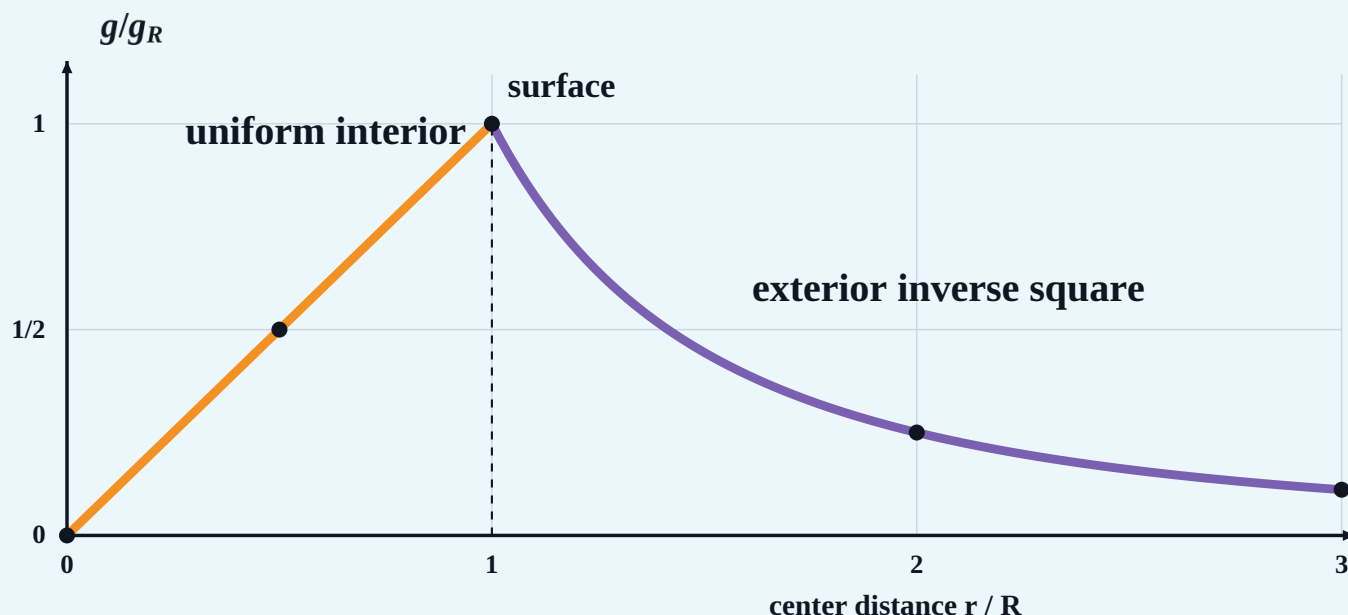
$$r = R/2 : M_{\text{enc}} = M/8$$

$$g = \frac{GM_{\text{enc}}}{r^2} = \frac{1}{2}g_R$$

For spherical symmetry, shells outside an interior point contribute zero net field there. In a uniform-density ball at $r = R/2$, only $M/8$ is enclosed, giving half the surface field. This is not a measured Earth-interior model.

Source: OpenStax, University Physics 1, 13.2

A Uniform Ball Has Two Connected Behaviors

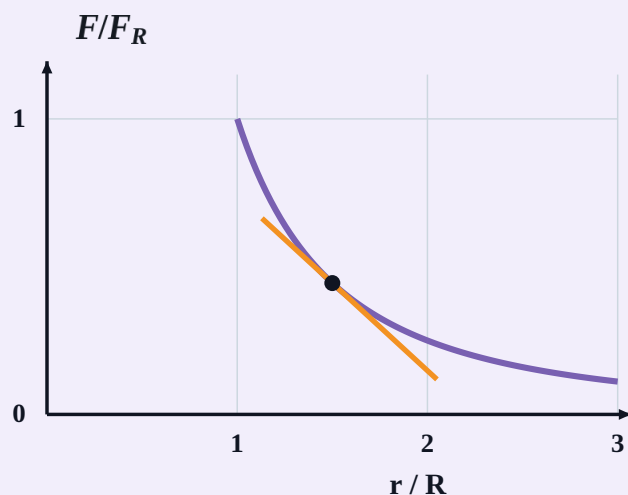


Inside the uniform ball, $g/g_R = r/R$. Outside, $g/g_R = (R/r)^2$. At the center the field is zero by symmetry—not by inserting $r = 0$ into the exterior quotient. The field is continuous at the surface.

Source: OpenStax, University Physics 1, 13.2

The Same Distances Lead Further

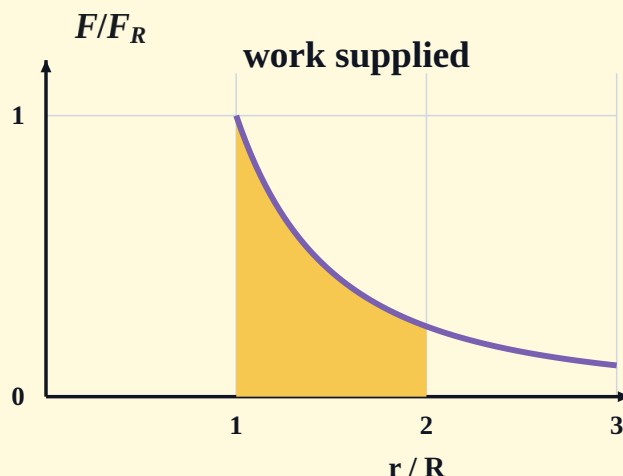
Slope: How Fast Force Changes



$$\frac{dF}{dr} = -\frac{2GMm}{r^3}$$

With masses fixed, the negative derivative means the exterior force decreases outward. The orange tangent gives a local change estimate, not the entire curve.

Area: Work Against Gravity

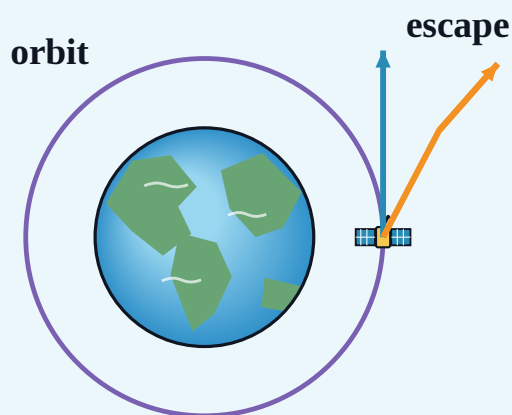


$$\Delta U = GMm \left(\frac{1}{r_1} - \frac{1}{r_2} \right)$$

For slow outward motion from r_1 to r_2 , the physical area under force magnitude versus distance is energy supplied. This graph uses normalized axes; $r_1 = R$ and $r_2 = 2R$.

Source: OpenStax, University Physics 1, 13.3

Orbit Speed & Escape Speed



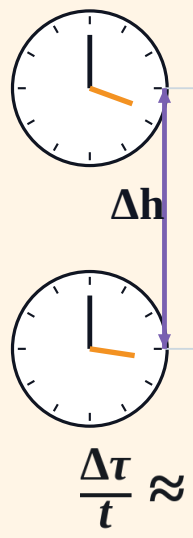
$$v_{\text{esc}} = \sqrt{2} v_{\text{circular}}$$

At the same r , ideal escape speed is $\sqrt{2}$ times circular speed. Assume an isolated spherical source, a tiny test mass, no drag, and no further thrust. Paths are schematic.

Source: OpenStax, University Physics 1, 13.3

Height Also Changes Clock Rate

higher: faster



$$\frac{\Delta \tau}{t} \approx \frac{g \Delta h}{c^2}$$

For otherwise comparable stationary clocks near Earth, the higher clock runs slightly faster from gravity alone. This is a weak-field, small-height approximation; dial differences are exaggerated.

Source: NIST, Atomic Clocks and Relativity (2010)