

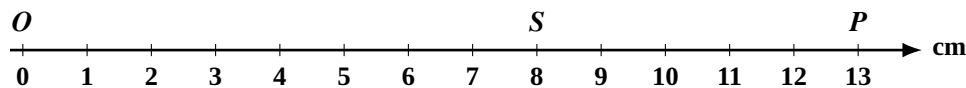
FREE MATH TUTOR / OBSERVE • MEASURE • EXPLAIN

Student Exercises / Read & Measure

Questions 1–12 / Identify the endpoints before doing arithmetic

For exercises that name the classroom Earth, use $R = 6,400$ km. Treat every body as spherical unless stated otherwise. Label units and show your reasoning. A notebook or the back of the sheet can hold longer sketches and calculations.

1. Complete the descriptions: R is measured from the body's center to its _____. h runs from the surface to the exterior _____. r runs from the _____ to that point.
2. Draw a circle for a spherical body. Label its center O , a surface point S , and an exterior point P on the same radial line. Add three separate arrows labeled R , h , and r . Which two lengths add to the third?
3. A model ball has radius $R = 5$ cm. A marker is $h = 3$ cm above its surface. Find r .
4. A body has radius $R = 6$ cm. The observation point is $r = 10$ cm from its center. Find h .
5. An observer is $r = 12$ cm from a model's center and $h = 4$ cm above its surface. Find the model's radius.
6. A point is exactly on the classroom Earth's surface. What are h and r ? Does this contradict the statement that R "need not equal" r ? Explain.
7. A probe is 100 km above the classroom Earth's surface. Find its center distance. Then state which number belongs in the denominator of the gravitational force law.
8. A probe is 7,000 km from the classroom Earth's center. Find its altitude.
9. A spherical body has $R = 10$ cm. Classify each point as *inside*, *on the surface*, or *outside*: (a) $r = 6$ cm; (b) $r = 10$ cm; (c) $r = 14$ cm.
10. A model's *diameter* is 20 cm. A marker is 7 cm beyond its surface. Find both R and r . Explain why $20 + 7$ is not the correct center distance.
11. Convert 6,800 km to meters. Write the result both as an ordinary whole number and in scientific notation.
12. Use the marked radial number line below. Find OS , SP , and OP . Match them to R , h , and r , and write an addition equation.



Explain in one sentence: Why can the body stay exactly the same while r changes?

Student Exercises / Compare & Calculate

Questions 13–24 / Use fractions, squares, and a correctly labeled distance

Use $q = F/F_R = g/g_R = (R/r)^2$ for exterior points. Hold the body and the test mass fixed unless a question says otherwise. For the classroom Earth, use $R = 6,400$ km and $g_R = 9.80$ m/s².

13. An object's gravitational force at the surface is 80 N. What is the force when its center distance becomes $2R$? Show the fraction used.
14. An object's surface gravitational force is 90 N. Find the force at a center distance of $3R$.
15. A satellite has altitude $h = R$. Express its center distance in terms of R , then find the fraction of surface gravity at that location.
16. A satellite has altitude $h = 3R$. Express r in terms of R . What fraction of surface gravity remains?
17. For a spherical body with $R = 10$ km, complete the table. Leave exact force fractions where possible.

Height h (km)	Center distance r (km)	F/F_R
0	_____	_____
10	_____	_____
20	_____	_____
30	_____	_____

18. Compare two height changes: (a) $h : 0 \rightarrow R$; (b) $h : R \rightarrow 2R$. Which one doubles the *center distance*? Find the new-to-old force ratio for each change.
19. For the classroom Earth at $h = 400$ km, find r , simplify R/r , and square that fraction to obtain q .
20. Use your result from Question 19 to find g at that height and the gravitational force on a 50 kg object. Round g to two decimal places and the force to the nearest newton.
21. A learner uses $r = 400$ km in Question 19. By what factor is the resulting force too large? Compare the incorrect formula with the correct formula.
22. A learner writes $(R + h)^2 = R^2 + h^2$. Identify the missing term. Then compare the two sides using $R = 3$ and $h = 4$.
23. An observation sphere expands from radius $2R$ to $3R$ around the same body. By what factor does its area change? By what factor does the gravitational field magnitude change?
24. Find the altitude above the classroom Earth where gravity is one fourth of its surface value. Give *both* the center distance and altitude.

Error check: A correct answer should identify whether a number is R , r , or h . Writing a number without naming the distance can hide a mistake.

Student Exercises / Investigate the Model

Questions 25–36 / Connect geometry, density, coordinates, and change

Use the spherical Newtonian model. For Questions 26–27, use the stated reference radii rather than the rounded classroom Earth's 6,400 km. Keep at least four significant digits until the final step.

25. At what altitude above the classroom Earth is gravity half its surface value? Use $R = 6,400$ km. Give an exact expression and a result rounded to the nearest kilometer.
26. Put an observation point 100 km above each body. Use $R_E = 6,371$ km, $R_M = 1,740$ km, and $R_S = 700,000$ km. Find each center distance and each ratio g/g_R . Which loses the largest *fraction* of its own surface gravity?
27. The Earth–Moon center separation is 384,400 km. Using the radii in Question 26, find the gap between the facing surfaces. Which distance belongs in $GM_E M_M / r^2$?
28. Two solid spherical laboratory balls have radii 2 cm and 3 cm. The gap between their facing surfaces is 5 cm. Find their center separation, first in centimeters and then in meters.
29. A uniform ball has $R = 3$ m and density $\rho = 2 \text{ kg/m}^3$. Find its volume and mass, leaving π in the answers. At $r = 6$ m, what fraction of its surface gravitational field remains?
30. A spherical body's radius doubles while its average density stays the same. Find the factors by which its volume, mass, and surface gravitational field change.
31. A different body's radius doubles while its total mass stays the same. What happens to its surface gravitational field? Why does this differ from Question 30?
32. A body's radius doubles at fixed density. An observer stays at the same center distance, equal to five times the *original* radius. The observer remains outside. By what factor does the field at the observer change? Why is the answer not the same as the surface comparison in Question 30?
33. Inside a uniform-density ball, let $r = R/2$. Find the enclosed mass fraction and the gravitational-field fraction. What is the field at the center? State why the exterior formula cannot be used there.
34. In a flat coordinate diagram, a spherical body's center is $(0, 0)$, its radius is 6 length units, and the observation point is $P = (6, 8)$. Use $r = \sqrt{x^2 + y^2}$ to find r , then find h and g/g_R . Is the coordinate $y = 8$ equal to the radial altitude?
35. Starting with $F(h) = GMm(R + h)^{-2}$, calculate dF/dh , holding G, M, m, R fixed. Explain the meaning of its sign for an exterior point.
36. At $h = 0.01R$, compare the exact ratio $q = (1 + h/R)^{-2}$ with the approximation $q \approx 1 - 2h/R$. State the exact and approximate percentage decreases from surface gravity.

Closing argument: Explain why “double the radius” can lead to one fourth, twice, or eight times the field, depending on what is held fixed and where the observation is made.