FREE MATH TUTOR / OBSERVE • MEASURE • EXPLAIN

Why R Is Not Always r

Radius, Center Distance & Height / A companion to page 13

> ***"The physical body's radius R need not equal the observer's center distance r."***[1]
> **Meaning: R tells how large the body is. r tells how far the measurement point is from that body's center.**

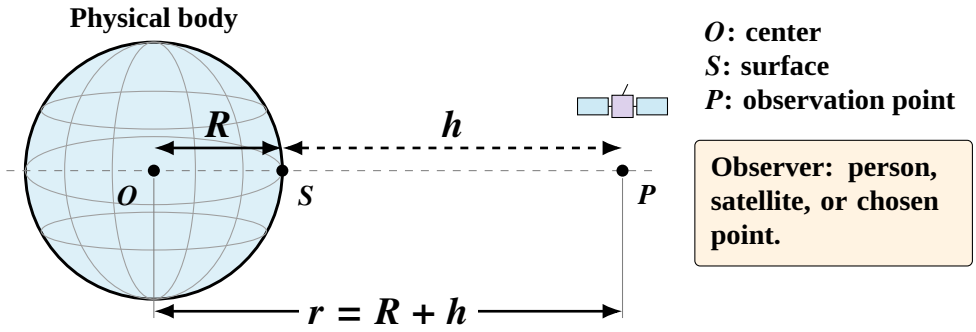

Figure 1. The radius and altitude are consecutive parts of the center distance. The diagram is schematic; the satellite marks point P, not a size drawn to scale.

Symbol	Measure from… to…	Its job
R	Center O to surface S	Describes the spherical body's size.
h	Surface S to point P, radially	Describes height above that surface.
r	Center O to point P	Goes into the gravitational denominator.

$$\boxed{r = R + h} \qquad \boxed{h = r - R}$$

For an exterior point, $h \geq 0$. If the point is exactly on the spherical surface, $h = 0$, so $r = R$. Above the surface, $h > 0$, so $r > R$. "Need not equal" does not mean "can never equal."

A number example

A rounded classroom Earth has $R = 6{,}400\,\text{km}$. A satellite $400\,\text{km}$ above its surface has

$$r = 6{,}400 + 400 = 6{,}800\,\text{km}, \qquad \textbf{not } 400\,\text{km}.$$

In $F = GMm/r^2$, the distance starts at the center, not at the ground. For a small object, its observation point is taken at its center of mass.[2,3]

The same force law, written using height

$$\boxed{F = \frac{GMm}{r^2} = \frac{GMm}{(R+h)^2}}$$

This is a substitution, not a new law. At the surface, $h = 0$, but the denominator is R^2, not zero. The surface is not the center.

Source notes: **[1]** ***Gravity: One Formula, Three Worlds*****, p. 13. The center-distance convention and exterior spherical model are supported by [2,3]. Full references are on p. 14.**

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One Body, Many Possible Locations

Article / Reading the sentence as a physical and mathematical idea

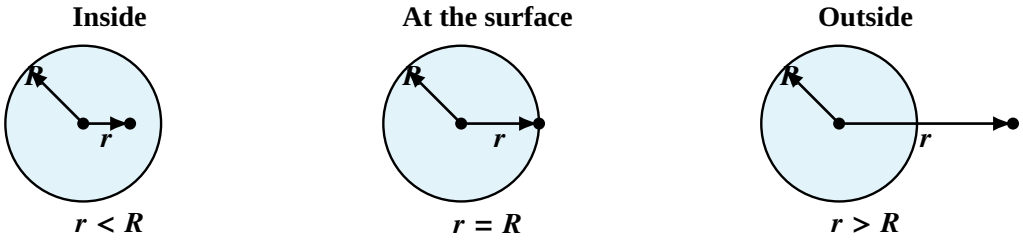

Figure 2. The body keeps the same radius in all three drawings. Only the observation point changes. The inside case is a mathematical location, not a claim that a person could stand inside solid rock.

A radius belongs to a body

Imagine a round ball on a table. Mark its center on a drawing, and measure to the ball's edge. That measurement is its radius. Now place a small marker somewhere beyond the edge. Moving the marker farther away does not make the ball larger. Its radius stays the same, although the center-to-marker distance changes.

That is the distinction between R and r. Capital R describes the physical object. Lowercase r describes a chosen position relative to its center. The letters resemble one another because both measurements are radial lengths, but they answer different questions.

A distance belongs to a location

The phrase "observer's center distance" means the distance *from the body's center to the observer's location*. It does not mean the observer's own body radius. In a force calculation for two separated spherical objects, it means the distance between their centers of mass.[2]

For an exterior point on a radial line, the trip from center to observer can be split into two consecutive pieces: center to surface, then surface to observer. Adding these pieces gives $r = R + h$. The height h is only the second piece. It cannot replace the entire distance r.

The equation can also be read backward. If you know where the observer is and how large the body is, subtract: $h = r - R$. A center distance of $10\,\mathrm{cm}$ from a body of radius $6\,\mathrm{cm}$ means a height of $4\,\mathrm{cm}$, not $10\,\mathrm{cm}$.

Equality has a special meaning

The word "need" matters. The original sentence does not forbid $r = R$. Equality identifies one particular location: the surface. At that location, the segment representing altitude has length zero.

Outside, $r > R$. Inside, $r < R$. At the center itself, $r = 0$. These statements classify positions; they do not, by themselves, tell us which gravitational formula is valid at every position.

The denominator is about separation

Newton's exterior spherical model uses $F = GMm/r^2$. Holding both masses fixed while changing r changes the predicted force. A greater center distance produces a smaller force. The body's physical radius does not grow merely because the observation point moves away.[2,3]

This is why a satellite's altitude cannot be used by itself in the denominator. At zero altitude, the satellite's location would be at the surface, not at Earth's center. The relevant center distance would still be approximately one Earth radius.

An imaginary sphere is not more planet

Now imagine placing a sphere around the body so that the observer lies on it. That surrounding sphere has radius r; the material body has radius R. The surrounding sphere is a measurement surface, not necessarily a shell made of matter. Enlarging it does not add mass to the planet.

The useful habit is to label *endpoints before numbers*. Ask, "Where does this measurement begin, and where does it end?" Then choose the equation. A correctly labeled picture often prevents an error before any arithmetic begins.

Read the sentence in everyday language: "The distance from the center to the edge of the object is not necessarily the same as the distance from the center to the place where we are measuring gravity."

Source notes: [1] supplies the original distinction. [2,3] support the Newtonian center-distance convention and the exterior/inside boundary. Analogies, numerical examples, and explanatory diagrams are developed here.

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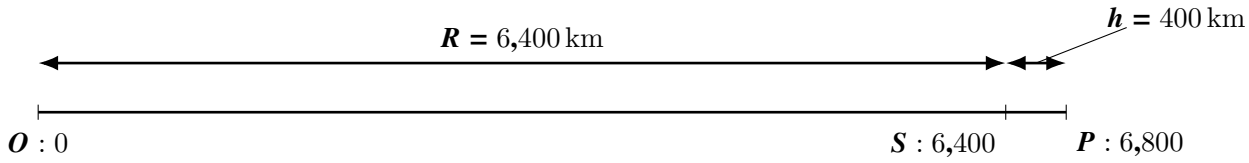
From Height to Gravitational Force

A worked example / Measure first, substitute second

Our rounded classroom Earth

Use $R = 6{,}400\,\text{km}$ and surface gravitational acceleration $g_R = 9.80\,\text{m/s}^2$. These define an approximate spherical, nonrotating model, not a precision Earth reference. A $50\,\text{kg}$ object is at height $h = 400\,\text{km}$. Find its center distance, gravitational acceleration, and gravitational force.

1 / Add the two consecutive distances



$$r = R + h = 6{,}400 + 400 = 6{,}800\,\text{km} = 6.8 \times 10^6\,\text{m}.$$

2 / Compare with gravity at the surface

For the same source mass M, the same test mass m, and an exterior observation point,

$$\frac{F(h)}{F_R} = \frac{g(h)}{g_R} = \frac{GM/(R+h)^2}{GM/R^2} = \left(\frac{R}{R+h}\right)^2.$$

Substitute the measurements. Kilometers cancel because this is a ratio of lengths:

$$q = \left(\frac{6{,}400}{6{,}800}\right)^2 = \left(\frac{16}{17}\right)^2 = \frac{256}{289} \approx 0.8858.$$

The field is about 88.58% of its surface value: a decrease of about 11.42%.

3 / Turn the fraction into acceleration and force

$$g(h) = 9.80\left(\frac{256}{289}\right) \approx 8.68\,\text{m/s}^2, \qquad F = mg \approx 434\,\text{N}.$$

The surface force on the same mass is $F_R = 50(9.80) = 490\,\text{N}$. Both force values describe gravitational attraction, directed toward the center in this model.

Why using altitude alone fails

Putting $h = 400\,\text{km}$ into the denominator in place of $r = 6{,}800\,\text{km}$ would make the predicted force too large by

$$\frac{F_{\text{wrong}}}{F_{\text{correct}}} = \frac{GMm/h^2}{GMm/r^2} = \left(\frac{6{,}800}{400}\right)^2 = 17^2 = 289.$$

The denominator would describe the wrong location. This is a geometry error, not merely a rounding error.

Two checks that protect the calculation

Units: Ratios can use kilometers consistently. Direct substitution with G in SI units requires meters, kilograms, and seconds. Also, $1\,\text{km}^2 = 10^6\,\text{m}^2$, not $10^3\,\text{m}^2$.

Weightlessness: A freely orbiting object can have zero apparent weight while still experiencing substantial gravity. Floating in orbit does not mean $g = 0$.[8]

Source notes: **Exterior gravity and altitude: [2,3]. Free fall and apparent weight: [8]. All numbers on this page are calculated from the stated classroom model.**

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Fractions, Squares & Distance

The inverse-square pattern / The fraction changes when the location changes

For one fixed body and the same test mass, define the fraction of surface gravity as

$$q = \frac{g}{g_R} = \frac{F}{F_R} = \left(\frac{R}{r}\right)^2 = \frac{1}{(1 + h/R)^2}, \qquad r \geq R.$$

The fraction is dimensionless: a ratio of two accelerations, two forces, or two squared lengths has no remaining units.

h/R	r/R	q
0	1	1
1/2	3/2	4/9
1	2	1/4
2	3	1/9
3	4	1/16

At height R, the center distance is $2R$. At height $2R$, it is $3R$. The columns cannot be interchanged.

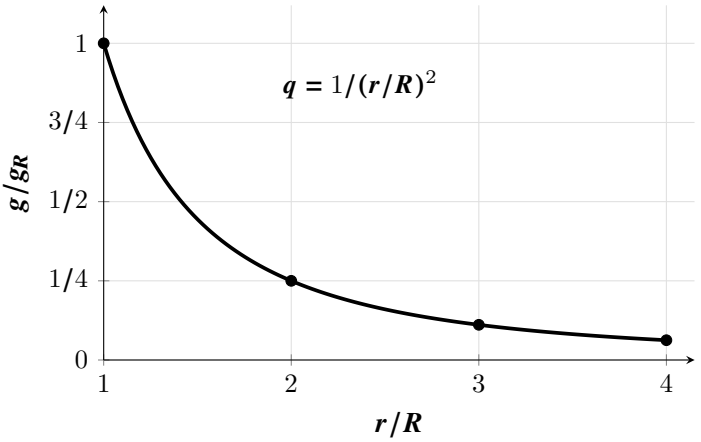

Figure 3. This graph describes points on or outside one spherical body. It is not a graph of the field inside the body.

Double which distance?

Doubling r gives one fourth the force. Doubling h does *not* generally do that. If height changes from R to $2R$, then center distance changes from $2R$ to $3R$:

$$\frac{F(2R \text{ in height})}{F(R \text{ in height})} = \left(\frac{2R}{3R}\right)^2 = \frac{4}{9}, \quad \text{not } \frac{1}{4}.$$

A fraction and a rational expression

With G, M, and m fixed, GMm/r^2 has the algebraic form of a rational function: a quotient of polynomials in r. This does not claim that every measured force is an exact rational number, which specifically means a ratio of integers. The denominator must be nonzero, and the exterior physical model has the stronger restriction $r \geq R > 0$.

Keep the entire sum inside the square:

$$(R + h)^2 = R^2 + 2Rh + h^2 \neq R^2 + h^2 \quad \text{in general.}$$

The terms R and r cannot simply be canceled as though they were the same variable.

Work backward from a desired fraction

For $0 < q \leq 1$, solve for the exterior height:

$$h = R\left(\frac{1}{\sqrt{q}} - 1\right).$$

Half the surface gravity occurs at $h = (\sqrt{2} - 1)R \approx 0.4142R$, not at R. One fourth occurs at $h = R$, where $r = 2R$.

Source notes: **The physical inverse-square model is supported by [2,3]. The fraction table, graph, algebraic classification, expansion, and rearrangements are derived here.**

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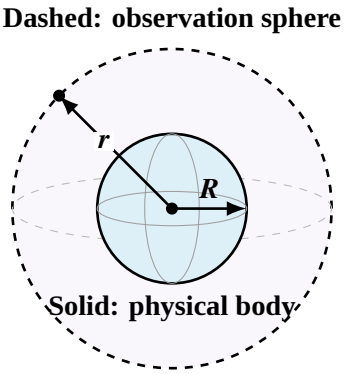
Two Radii, Two Jobs in Geometry

The material body and the surrounding measurement sphere



Use R to describe the body

Its volume and average density are

$$V_{\text{body}} = \frac{4}{3}\pi R^3, \qquad \bar{\rho} = \frac{M}{V_{\text{body}}}.$$

Therefore,

$$\boxed{M = \frac{4}{3}\pi\bar{\rho}R^3}.$$

This last equation uses *average* density and does not require every layer to have the same density. Uniform density becomes important when we calculate the enclosed mass at an interior point.

Figure 4. The solid body's size is R; the dashed measurement sphere's size is r. Moving the dashed boundary does not add matter to the solid body.

Put density into the exterior gravity equation

For a spherically symmetric body and $r \geq R$,

$$g(r) = \frac{GM}{r^2} = \frac{4\pi G\bar{\rho}}{3}\frac{R^3}{r^2}.$$

Only at the surface can we set $r = R$ and obtain

$$\boxed{g_R = \frac{4\pi G\bar{\rho}}{3}R}.$$

The appearance of both R and r is intentional: one determines the body's volume; the other identifies the observation location.

Use r to describe the surrounding sphere

A sphere centered on the body and passing through the observer has area $A_s = 4\pi r^2$. For the exterior spherical field,

$$gA_s = 4\pi GM, \qquad F = \frac{4\pi GMm}{A_s}.$$

Doubling r multiplies that area by four and makes g one fourth as large. The equation is a geometric description of the field, not a claim that gravity is a substance that gets used up. Replacing r^2 by A_s *without* the factor 4π changes the law.[1,2]

The phrase "double the radius" needs a condition

Change	What is held fixed?	New g
Observer moves from r to $2r$	The same body, with the same M, R	$g/4$
Body's R doubles; compare surfaces	Total mass M	$g/4$
Body's R doubles; compare surfaces	Average density $\bar{\rho}$	$2g$
Body's R doubles; observer stays put	Density and exterior center distance r	$8g$

In the last row, the observer must remain outside the enlarged body. When density is fixed, the mass grows as R^3: doubling R gives eight times the mass.

Source notes: **Original geometric distinction: [1]. Spherical exterior gravity: [2,3]. Mean radius and bulk density conventions: [4]. Equations and scaling comparisons are derived here.**

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Sun, Earth & Moon

The same distinction works at three very different scales

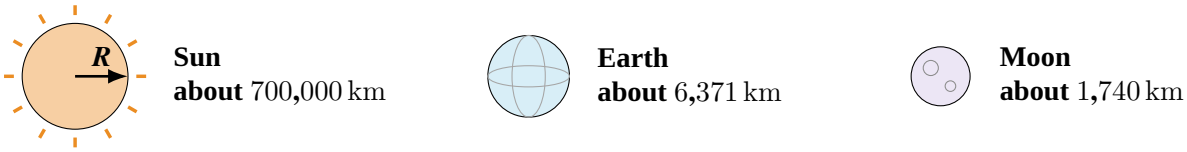

Figure 5. Reference radii from NASA and JPL, rounded for comparison; the pictures are not to scale. Earth's value is a mean radius. The Sun's visible "surface" is the photosphere, not solid ground.[4,5,6]

Equal altitude does not mean equal relative distance

Suppose a hypothetical observation point is $400\,\text{km}$ above each chosen reference surface. Use each body's own radius in $r = R + h$:

Body	R (km)	$r = R + 400$ (km)	$g/g_R = (R/r)^2$
Sun	700,000	700,400	0.9989
Earth	6,371	6,771	0.8853
Moon	1,740	2,140	0.6611

These are fractions of *each body's own* surface gravity, not a comparison of their absolute accelerations. A height of $400\,\text{km}$ is a much larger fraction of the Moon's radius than of the Sun's radius. The Sun row is an idealized spherical calculation that ignores atmospheric mass outside the chosen surface.

Equal relative height gives an equal fraction: at $h = R$ above *any* of these idealized spherical bodies, $r = 2R$, so $g/g_R = 1/4$. Equal fractions do not imply equal forces in newtons.

For two bodies, use center-to-center distance

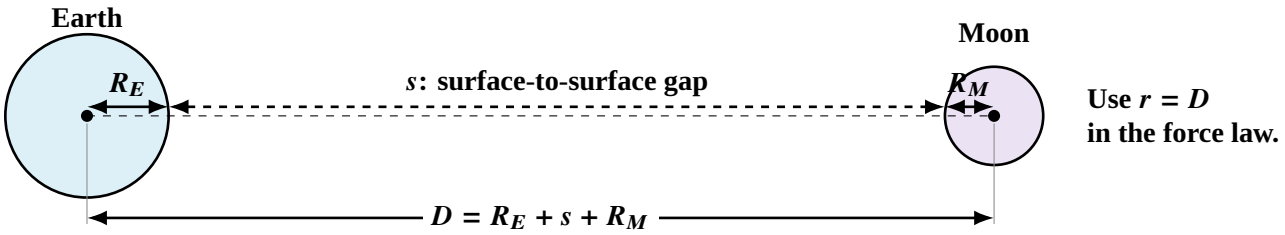

Figure 6. The surface gap and center separation are different lengths. The sizes and gap are schematic.

Using the Moon's approximate average center separation $D = 384{,}400\,\text{km}$,[5]

$$s = 384{,}400 - 6{,}371 - 1{,}740 = 376{,}289\,\text{km}.$$

The spherical Earth–Moon force calculation uses $F = GM_EM_M/D^2$, not the gap s. The distance varies during the orbit; this is a representative value, not a live measurement.

For the Sun's pull on Earth, the relevant r is their center separation, about 150 million kilometers, not the Sun's radius of about $700{,}000$ kilometers.[6]

Source notes: **Earth mean radius: [4]. Moon radius and average separation: [5]. Solar radius, photosphere, and approximate Earth separation: [6]. Center-to-center force convention: [2]. The table and surface gap are calculated here.**

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Inside the Body & Beyond

Why the choice of model matters as much as the choice of distance

Do not use all of M as a point mass inside the body

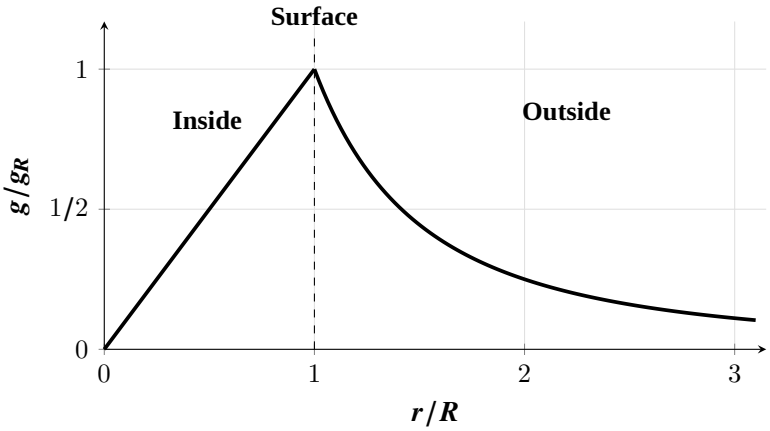
For a spherically symmetric body, an interior point at $0 < r < R$ responds to the mass $M_{\text{enc}}(r)$ enclosed within that radius. Spherical shells outside the point give zero net field there. Thus[3]

$$g(r) = \frac{GM_{\text{enc}}(r)}{r^2}, \quad 0 < r < R.$$

For a *uniform-density* ball, $M_{\text{enc}} = M(r/R)^3$. Therefore,

$$\boxed{\frac{g(r)}{g_R} = \begin{cases} r/R, & 0 \le r \le R, \\ (R/r)^2, & r \ge R. \end{cases}}$$

At the center the field is zero by symmetry; it is not found by substituting $r = 0$ into the exterior formula. Here g is a magnitude; at noncentral points the field points inward.



At $r = R/2$, a uniform ball has only $M/8$ enclosed. Consequently,

$$\frac{g}{g_R} = \frac{1/8}{(1/2)^2} = \frac{1}{2}.$$

Using the entire M would incorrectly predict $4g_R$.

Figure 7. A uniform-ball model, not a measured profile of Earth's interior. Real Earth is layered and nonuniform; its interior field is not generally a straight line.[3]

A closer look at how fast the force changes

Holding G, M, m, R fixed and differentiating with respect to height gives

$$F(h) = GMm(R+h)^{-2}, \qquad \boxed{\frac{dF}{dh} = -\frac{2GMm}{(R+h)^3} < 0}.$$

For heights much smaller than the body's radius, the first-order approximation is

$$\frac{g(h)}{g_R} = \left(1 + \frac{h}{R}\right)^{-2} \approx 1 - \frac{2h}{R}, \qquad h/R \ll 1.$$

Where relativity fits

General relativity describes gravity through spacetime geometry and free fall, rather than only a Newtonian attraction force.[8] This lesson's $r = R + h$ uses ordinary straight-line spatial geometry. In strong-gravity relativistic models, radial coordinates and physical distance measurements require more careful definitions; the elementary addition rule should not be treated as a universal statement about every coordinate system.

Source notes: **Spherical shells, enclosed mass, and the uniform-interior distinction: [3]. General-relativistic viewpoint: [8]. Coordinate, derivative, and approximation formulas are derived here.**

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Student Exercises / Read & Measure

Questions 1–12 / Identify the endpoints before doing arithmetic

For exercises that name the classroom Earth, use $R = 6{,}400\,\text{km}$. Treat every body as spherical unless stated otherwise. Label units and show your reasoning. A notebook or the back of the sheet can hold longer sketches and calculations.

1. Complete the descriptions: R is measured from the body's center to its _____. h runs from the surface to the exterior _____. r runs from the _____ to that point.

2. Draw a circle for a spherical body. Label its center O, a surface point S, and an exterior point P on the same radial line. Add three separate arrows labeled R, h, and r. Which two lengths add to the third?

3. A model ball has radius $R = 5\,\text{cm}$. A marker is $h = 3\,\text{cm}$ above its surface. Find r.

4. A body has radius $R = 6\,\text{cm}$. The observation point is $r = 10\,\text{cm}$ from its center. Find h.

5. An observer is $r = 12\,\text{cm}$ from a model's center and $h = 4\,\text{cm}$ above its surface. Find the model's radius.

6. A point is exactly on the classroom Earth's surface. What are h and r? Does this contradict the statement that R "need not equal" r? Explain.

7. A probe is $100\,\text{km}$ above the classroom Earth's surface. Find its center distance. Then state which number belongs in the denominator of the gravitational force law.

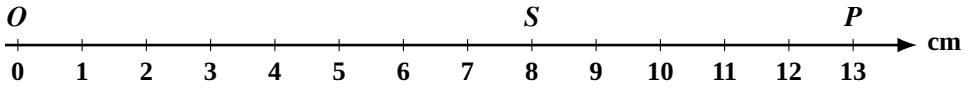
8. A probe is $7{,}000\,\text{km}$ from the classroom Earth's center. Find its altitude.

9. A spherical body has $R = 10\,\text{cm}$. Classify each point as *inside*, *on the surface*, or *outside*: (a) $r = 6\,\text{cm}$; (b) $r = 10\,\text{cm}$; (c) $r = 14\,\text{cm}$.

10. A model's *diameter* is $20\,\text{cm}$. A marker is $7\,\text{cm}$ beyond its surface. Find both R and r. Explain why $20 + 7$ is not the correct center distance.

11. Convert $6{,}800\,\text{km}$ to meters. Write the result both as an ordinary whole number and in scientific notation.

12. Use the marked radial number line below. Find OS, SP, and OP. Match them to R, h, and r, and write an addition equation.



Explain in one sentence: Why can the body stay exactly the same while r changes?

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Student Exercises / Compare & Calculate

Questions 13–24 / Use fractions, squares, and a correctly labeled distance

Use $q = F/F_R = g/g_R = (R/r)^2$ for exterior points. Hold the body and the test mass fixed unless a question says otherwise. For the classroom Earth, use $R = 6{,}400\,\mathrm{km}$ and $g_R = 9.80\,\mathrm{m/s^2}$.

13. An object's gravitational force at the surface is $80\,\mathrm{N}$. What is the force when its center distance becomes $2R$? Show the fraction used.

14. An object's surface gravitational force is $90\,\mathrm{N}$. Find the force at a center distance of $3R$.

15. A satellite has altitude $h = R$. Express its center distance in terms of R, then find the fraction of surface gravity at that location.

16. A satellite has altitude $h = 3R$. Express r in terms of R. What fraction of surface gravity remains?

17. For a spherical body with $R = 10\,\mathrm{km}$, complete the table. Leave exact force fractions where possible.

Height h (km)	Center distance r (km)	F/F_R
0	_____	_____
10	_____	_____
20	_____	_____
30	_____	_____

18. Compare two height changes: (a) $h : 0 \to R$; (b) $h : R \to 2R$. Which one doubles the *center distance*? Find the new-to-old force ratio for each change.

19. For the classroom Earth at $h = 400\,\mathrm{km}$, find r, simplify R/r, and square that fraction to obtain q.

20. Use your result from Question 19 to find g at that height and the gravitational force on a $50\,\mathrm{kg}$ object. Round g to two decimal places and the force to the nearest newton.

21. A learner uses $r = 400\,\mathrm{km}$ in Question 19. By what factor is the resulting force too large? Compare the incorrect formula with the correct formula.

22. A learner writes $(R + h)^2 = R^2 + h^2$. Identify the missing term. Then compare the two sides using $R = 3$ and $h = 4$.

23. An observation sphere expands from radius $2R$ to $3R$ around the same body. By what factor does its area change? By what factor does the gravitational field magnitude change?

24. Find the altitude above the classroom Earth where gravity is one fourth of its surface value. Give *both* the center distance and altitude.

Error check: A correct answer should identify whether a number is R, r, or h. Writing a number without naming the distance can hide a mistake.

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Student Exercises / Investigate the Model

Questions 25–36 / Connect geometry, density, coordinates, and change

Use the spherical Newtonian model. For Questions 26–27, use the stated reference radii rather than the rounded classroom Earth's $6{,}400\,\mathrm{km}$. Keep at least four significant digits until the final step.

25. At what altitude above the classroom Earth is gravity half its surface value? Use $R = 6{,}400\,\mathrm{km}$. Give an exact expression and a result rounded to the nearest kilometer.

26. Put an observation point $100\,\mathrm{km}$ above each body. Use $R_E = 6{,}371\,\mathrm{km}$, $R_M = 1{,}740\,\mathrm{km}$, and $R_S = 700{,}000\,\mathrm{km}$. Find each center distance and each ratio g/g_R. Which loses the largest *fraction* of its own surface gravity?

27. The Earth–Moon center separation is $384{,}400\,\mathrm{km}$. Using the radii in Question 26, find the gap between the facing surfaces. Which distance belongs in GM_EM_M/r^2?

28. Two solid spherical laboratory balls have radii $2\,\mathrm{cm}$ and $3\,\mathrm{cm}$. The gap between their facing surfaces is $5\,\mathrm{cm}$. Find their center separation, first in centimeters and then in meters.

29. A uniform ball has $R = 3\,\mathrm{m}$ and density $\rho = 2\,\mathrm{kg/m^3}$. Find its volume and mass, leaving π in the answers. At $r = 6\,\mathrm{m}$, what fraction of its surface gravitational field remains?

30. A spherical body's radius doubles while its average density stays the same. Find the factors by which its volume, mass, and surface gravitational field change.

31. A different body's radius doubles while its total mass stays the same. What happens to its surface gravitational field? Why does this differ from Question 30?

32. A body's radius doubles at fixed density. An observer stays at the same center distance, equal to five times the *original* radius. The observer remains outside. By what factor does the field at the observer change? Why is the answer not the same as the surface comparison in Question 30?

33. Inside a uniform-density ball, let $r = R/2$. Find the enclosed mass fraction and the gravitational-field fraction. What is the field at the center? State why the exterior formula cannot be used there.

34. In a flat coordinate diagram, a spherical body's center is $(0, 0)$, its radius is 6 length units, and the observation point is $P = (6, 8)$. Use $r = \sqrt{x^2 + y^2}$ to find r, then find h and g/g_R. Is the coordinate $y = 8$ equal to the radial altitude?

35. Starting with $F(h) = GMm(R + h)^{-2}$, calculate dF/dh, holding G, M, m, R fixed. Explain the meaning of its sign for an exterior point.

36. At $h = 0.01R$, compare the exact ratio $q = (1 + h/R)^{-2}$ with the approximation $q \approx 1 - 2h/R$. State the exact and approximate percentage decreases from surface gravity.

Closing argument: Explain why "double the radius" can lead to one fourth, twice, or eight times the field, depending on what is held fixed and where the observation is made.

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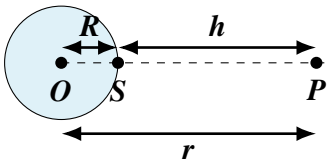
Worked Answers / Measurements & Fractions

Questions 1–18 / Match each number to a segment before using it

1. Surface; observation point; center. The segments have endpoints $OS = R$, $SP = h$, and $OP = r$.

2. A correct sketch puts O, S, P in that order on the same outward radial line. The first two lengths add to the whole: $R + h = r$.



3. $r = R + h = 5 + 3 = \boxed{8\,\text{cm}}$. The $3\,\text{cm}$ is measured from the surface, not from the center.

4. $h = r - R = 10 - 6 = \boxed{4\,\text{cm}}$. Subtract the part inside the body from the whole center distance.

5. $R = r - h = 12 - 4 = \boxed{8\,\text{cm}}$. Check: $8 + 4 = 12$.

6. $\boxed{h = 0}$, and $\boxed{r = 6{,}400\,\text{km} = R}$. There is no contradiction. The surface is the location where the two lengths are equal. "Need not" allows equality but does not require it everywhere.

7. $r = 6{,}400 + 100 = \boxed{6{,}500\,\text{km}}$. This is the distance to square. In direct SI substitution, use $6.5 \times 10^6\,\text{m}$, not $100\,\text{km}$.

8. $h = 7{,}000 - 6{,}400 = \boxed{600\,\text{km}}$. The probe is $7{,}000\,\text{km}$ from the center but only $600\,\text{km}$ above the surface.

9. (a) Inside, since $6 < 10$. (b) On the surface, since $10 = 10$. (c) Outside, since $14 > 10$. The radius itself stays $10\,\text{cm}$ in all three cases.

10. $R = 20/2 = \boxed{10\,\text{cm}}$, and $r = 10 + 7 = \boxed{17\,\text{cm}}$. Adding the entire diameter would start at the opposite surface, not at the center.

11. $6{,}800 \times 1{,}000 = \boxed{6{,}800{,}000\,\text{m}} = \boxed{6.8 \times 10^6\,\text{m}}$. The numerical value changes because the measurement unit changes; the actual distance does not.

12. $OS = 8\,\text{cm} = R$, $SP = 13 - 8 = 5\,\text{cm} = h$, and $OP = 13\,\text{cm} = r$. The addition equation is $\boxed{8 + 5 = 13}$.

13. $q = (R/2R)^2 = 1/4$. Thus $F = 80/4 = \boxed{20\,\text{N}}$. The center distance, not the height, was doubled.

14. $q = (R/3R)^2 = 1/9$. Therefore $F = 90/9 = \boxed{10\,\text{N}}$.

15. $r = R + R = \boxed{2R}$, so $\boxed{g/g_R = 1/4}$. One radius of altitude means two radii from the center.

16. $r = R + 3R = \boxed{4R}$, so $\boxed{g/g_R = 1/16}$.

17. First add $10\,\text{km}$ to each height. Then square the ratio $10/r$:

h (km)	r (km)	F/F_R
0	10	1
10	20	$1/4$
20	30	$1/9$
30	40	$1/16$

18. (a) $r : R \to 2R$, so the center distance doubles and the force ratio is $\boxed{1/4}$. (b) $r : 2R \to 3R$, so the center distance grows by $3/2$ and the force ratio is $\boxed{4/9}$. Doubling altitude does not generally double center distance.

One-sentence explanation: The physical radius stays fixed when the same body keeps its size, but the center distance changes whenever the observation point moves nearer to or farther from its center.

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Worked Answers / Forces & Three Worlds

Questions 19–28 / Preserve the whole denominator and the meaning of each distance

19. $r = 6{,}400 + 400 = 6{,}800\,\text{km}$. **Then**

$$\frac{R}{r} = \frac{6{,}400}{6{,}800} = \frac{16}{17}, \qquad \boxed{q = \frac{256}{289} \approx 0.885813}.$$

20. $g = 9.80(256/289) = 8.680968\ldots$, **giving** $\boxed{g \approx 8.68\,\text{m/s}^2}$. **With the unrounded value,** $F = 50g = 434.048\ldots$, **so** $\boxed{F \approx 434\,\text{N}}$. **The surface force is** $490\,\text{N}$.

21. The mistaken distance is 17 **times too small, so the inverse-square force is** 17^2 **times too large:**

$$\boxed{\frac{F_{\text{wrong}}}{F_{\text{correct}}} = \left(\frac{6{,}800}{400}\right)^2 = 289}.$$

22. The missing term is $\boxed{2Rh}$. **For** $R = 3$ **and** $h = 4$, **the actual square is** $(3+4)^2 = 49$, **while** $3^2 + 4^2 = 25$. **Adding** $2(3)(4) = 24$ **fixes the difference:** $9 + 24 + 16 = 49$.

23. The area grows by $(3R/2R)^2 = \boxed{9/4}$. **The field changes by the reciprocal factor,** $(2R/3R)^2 = \boxed{4/9}$. **The physical body and its mass have not changed.**

24. A fraction of $1/4$ **means** $R/r = 1/2$, **hence** $r = 2R$. **Therefore** $\boxed{r = 12{,}800\,\text{km}}$ **and** $\boxed{h = r - R = 6{,}400\,\text{km}}$.

25. Set $q = 1/2$ **and solve:**

$$\frac{1}{2} = \left(\frac{R}{R+h}\right)^2 \Longrightarrow R + h = \sqrt{2}R \Longrightarrow \boxed{h = 6{,}400(\sqrt{2} - 1)\,\text{km}}.$$

Numerically, $h = 2{,}650.9668\ldots\,\text{km}$, **so** $\boxed{h \approx 2{,}651\,\text{km}}$. **The center distance is about** $9{,}051\,\text{km}$; **it is not the same as the altitude.**

26. Use $r = R + 100\,\text{km}$, **then** $q = (R/r)^2$. **Values below are computed from the rounded reference radii given in the question.**

Body	r (km)	Fraction q	Decrease $100(1-q)$
Earth	6,471	0.9693	**about** 3.07%
Moon	1,840	0.8943	**about** 10.57%
Sun	700,100	0.9997	**about** 0.0286%

The Moon loses the largest fraction. Its $100\,\text{km}$ **altitude is the largest fraction of the body's radius. This conclusion compares percentage changes, not absolute gravitational accelerations.**

27. Subtract the two radii from the center separation:

$$\boxed{s = 384{,}400 - 6{,}371 - 1{,}740 = 376{,}289\,\text{km}}.$$

Use $\boxed{r = 384{,}400\,\text{km}}$ **in the two-body force law, or** $3.844 \times 10^8\,\text{m}$ **with SI** G. **The gap is useful geometry but not the force-law separation.**

28. The path from one center to the other consists of the first radius, the gap, and the second radius: $D = 2 + 5 + 3 =$ $\boxed{10\,\text{cm}}$ $=$ $\boxed{0.10\,\text{m}}$. **It is neither the** $5\,\text{cm}$ **gap nor the sum of the two diameters and the gap.**

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Worked Answers / Geometry & Change

Questions 29–36 / State what is fixed before comparing gravitational fields

29. $V = (4/3)\pi(3)^3 = \boxed{36\pi\ \mathrm{m}^3}$. The mass is $M = \rho V = 2(36\pi) = \boxed{72\pi\ \mathrm{kg}}$. At $r = 6\ \mathrm{m} = 2R$, the field fraction is $\boxed{(3/6)^2 = 1/4}$.

30. Volume is proportional to R^3, so it grows by $2^3 = \boxed{8}$. At fixed density, mass also grows by $\boxed{8}$. Surface field is GM/R^2, so its factor is $8/2^2 = \boxed{2}$.

31. The mass does not change, but the surface distance doubles. Therefore $\boxed{g_{\mathrm{new}}/g_{\mathrm{old}} = 1/4}$. Unlike Question 30, no factor of eight enters the numerator. In fact, the average density becomes one eighth of its old value.

32. Fixed density and double radius give $M_{\mathrm{new}} = 8M_{\mathrm{old}}$. Since the observation distance is unchanged, $\boxed{g_{\mathrm{new}}/g_{\mathrm{old}} = 8}$. The observer is at $5R_{\mathrm{old}} > 2R_{\mathrm{old}}$, so the exterior model still applies. Question 30 instead compared two different surface locations.

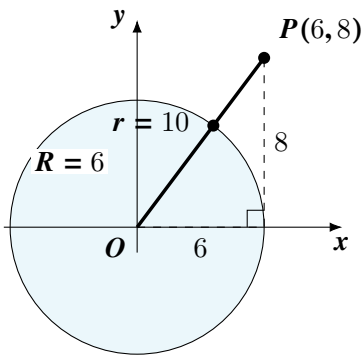
33. A uniform ball has $M_{\mathrm{enc}}/M = (r/R)^3 = \boxed{1/8}$. The field ratio is $(1/8)/(1/2)^2 = \boxed{1/2}$. At the center, $\boxed{g = 0}$ by symmetry. The exterior formula with the total M does not account for interior shell cancellation and is invalid at this location.

34. By the Pythagorean theorem,

$$r = \sqrt{6^2 + 8^2} = \sqrt{100} = \boxed{10}.$$

Thus $\boxed{h = 10 - 6 = 4}$ length units and

$$\boxed{\frac{g}{g_R} = \left(\frac{6}{10}\right)^2 = \frac{9}{25} = 0.36}.$$



The coordinate $y = 8$ is *not* the altitude. Altitude is measured radially from the spherical surface, along the line from the center to P.

35. Apply the power rule and chain rule:

$$\frac{dF}{dh} = GMm[-2(R+h)^{-3}]\frac{d(R+h)}{dh} = \boxed{-\frac{2GMm}{(R+h)^3}}.$$

For positive masses and an exterior point, the derivative is negative. Increasing altitude decreases the gravitational-force magnitude. With SI quantities, dF/dh has units of newtons per meter.

36. The exact fraction is $q = 1/(1.01)^2 = \boxed{0.980296\ldots}$. The approximation gives $q \approx 1 - 2(0.01) = \boxed{0.98}$. The exact decrease is $\boxed{1.9704\ldots\%}$; the approximate decrease is $\boxed{2.00\%}$. The approximation slightly overestimates the decrease by about 0.0296 percentage points.

Closing argument: The same words can describe different experiments. Double r around a fixed body: one fourth. Double R at fixed density and compare surfaces: twice. Double R at fixed density while keeping an exterior r fixed: eight times. Always name the fixed quantities and the observation location.

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Keep This Reference

Essential formulas, a hands-on investigation, and scientific sources

Question	Equation and condition
Where is the point?	$r = R + h$; $h = r - R$ for radial exterior height.
How strong is exterior gravity?	$F = GMm/r^2$; $g = GM/r^2$; spherical body, $r \geq R$.
What fraction of surface gravity?	$q = (R/r)^2 = (1 + h/R)^{-2}$.
What height gives fraction q?	$h = R(q^{-1/2} - 1)$, with $0 < q \leq 1$.
How much matter is in the body?	$V = (4/3)\pi R^3$; $M = \bar{\rho}V$.
What is the surrounding area?	$A_s = 4\pi r^2$; outside, $gA_s = 4\pi GM$.
What happens inside a uniform ball?	$M_{\text{enc}} = M(r/R)^3$; $g/g_R = r/R$.
How are two body centers separated?	$D = R_1 + s + R_2$; s is their facing-surface gap.

$G \approx 6.67430 \times 10^{-11}\ \text{m}^3\,\text{kg}^{-1}\,\text{s}^{-2}$. This is a measured constant, not an exact definition.[7] Here F and g mean magnitudes; gravitational acceleration points inward for the spherical cases discussed. g_R means the field evaluated at the physical surface $r = R$.

Hands-on investigation / The body stays, the marker moves

Draw a circle with radius $4\,\text{cm}$. Mark three points on one outward radial line at center distances 4, 8, and $12\,\text{cm}$. Measure their surface heights: 0, 4, and $8\,\text{cm}$. Then predict the gravity fractions: 1, $1/4$, and $1/9$. Ask a partner to point to the two equal lengths at the first location and the two unequal lengths at the others. This is a scale-model investigation, not an attempt to detect the tiny gravitational pull of the paper drawing.

Sources & Further Reading

[1] Free Math Tutor. *Gravity: One Formula, Three Worlds*. Page 13, "Circles, Spheres & Density." Original lesson supplied in this conversation; the quoted sentence and the R–r distinction are the starting point for this companion.

[2] OpenStax. *University Physics, Volume 1*, Section 13.1, "Newton's Law of Universal Gravitation." Center-to-center separation and the spherical-body model. Read the section.

[3] OpenStax. *University Physics, Volume 1*, Section 13.2, "Gravitation Near Earth's Surface." Altitude, spherical shells, enclosed mass, and uniform versus real planetary interiors. Read the section.

[4] NASA Jet Propulsion Laboratory, Solar System Dynamics. "Planetary Physical Parameters." Earth's mean radius and the meanings of mean radius and bulk density. Read the reference table.

[5] NASA Science. "Moon Facts," especially "Size and Distance." Rounded lunar radius and approximate mean Earth–Moon separation. Read Moon Facts.

[6] NASA Science. "Sun: Facts," especially "Size and Distance" and the discussion of the photosphere. Rounded solar radius and approximate Sun–Earth distance. Read Sun Facts.

[7] National Institute of Standards and Technology. *Fundamental Physical Constants—Extensive Listing*, 2022 CODATA adjustment, p. 1. Recommended value of G. Open the constants reference.

[8] Einstein Online, Max Planck Institute for Gravitational Physics. "Gravity: from weightlessness to curvature." Free fall, apparent weight, and the geometric view of gravitation. Read the article.

Reference pages checked September 21, 2026. Numerical results are rounded as indicated. The classroom Earth uses a deliberately rounded radius; it should not be confused with the NASA/JPL reference radius. Diagrams are original mathematical illustrations and are schematic unless a coordinate or scale construction is specified.