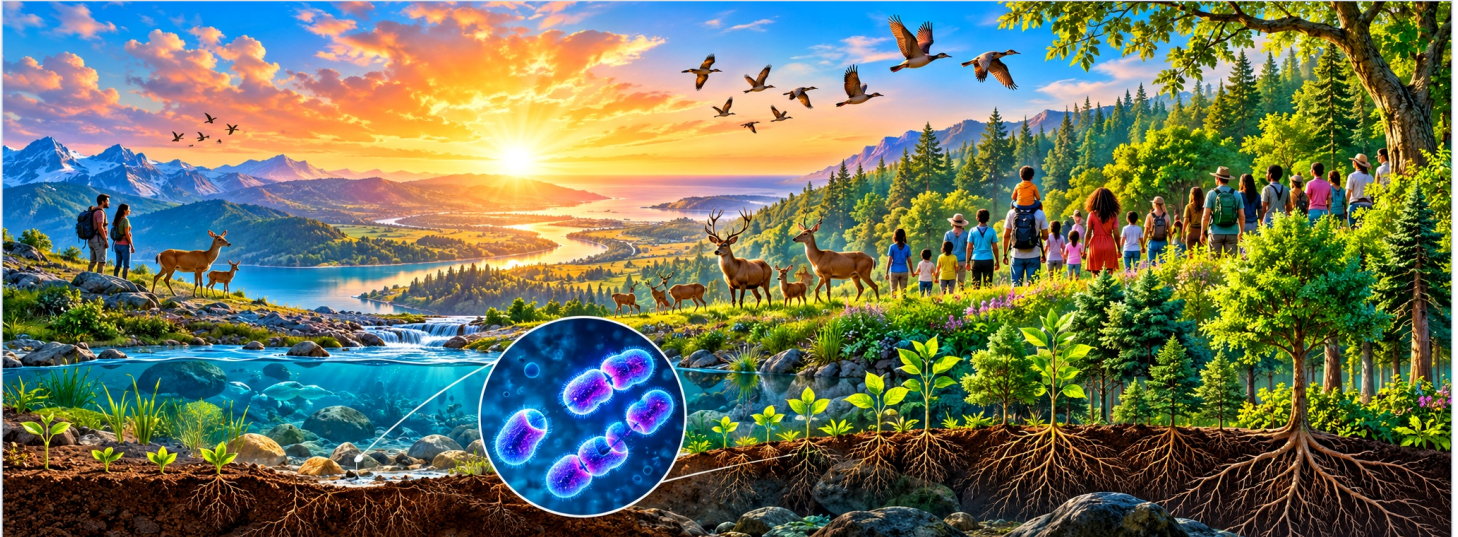


POPULATION GROWTH

FORMULAS FOR HUMANS & LIVING SYSTEMS

An Illustrated Mathematical Article, Definition Guide, Formula Table, and 1933-2033 Case Study



Population mathematics connects repeated multiplication to births, deaths, migration, resources, age structure, and interaction among species. The same core ideas can describe humans, animals, plants, microorganisms, cells, and ecological communities - but the correct model depends on the biology.

CORE FORMULA

$$P(t) = P_0(1 + r)^t$$

A discrete model with a constant proportional change per time step.

ESSENTIAL CORRECTION

$$2.71\% = 0.0271.$$

$$0.00271 = 0.271\%.$$

Euler's number $e = 2.71828\ldots$ is not a percent rate.

THE 100-YEAR EXPERIMENT

Start with 19,000,000 individuals in 1933. Hold each annual rate constant through 2033: $r = 0.001, 0.01, 0.1, 1.0$, and 10.0 . The resulting magnitudes reveal why a small change in rate produces a gigantic long-term change.

Teacher/Student Reference - September 24, 2026

From Repetition to Population: The Mathematics of Living Change

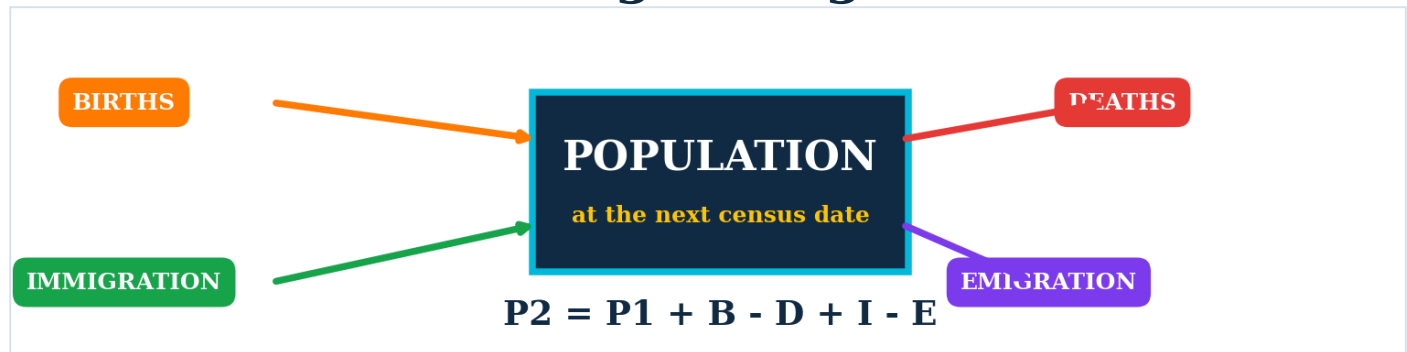


Figure 1. The demographic balancing equation accounts directly for the four flows that change a human population: births, deaths, immigration, and emigration.

A population is not merely a number. It is a number attached to a place, a time, and a definition of membership. A town's population may change because babies are born, residents die, people move in, or people move out. The U.S. Census Bureau organizes these flows with the demographic balancing equation, while professional projections often divide the population into age and sex groups and advance each cohort through time. The United Nations also uses cohort-component methods and separate assumptions about fertility, mortality, and international migration for long-range national projections.^{[1][2]}

The familiar model $P(t) = P_0(1 + r)^t$ begins with a simpler idea: in every equal time interval, the population changes by the same proportion. If there are more individuals now, the next period's numerical increase is larger even though the percentage stays the same. Repeated proportional change creates geometric or exponential growth. This is why compound interest, bacterial multiplication, and idealized population growth share related equations.

WHY AN EXPONENT APPEARS

One step gives $P_1 = P_0(1 + r)$. Two steps give $P_2 = P_0(1 + r)(1 + r) = P_0(1 + r)^2$. After t equal steps, the factor has been multiplied t times: $P_t = P_0(1 + r)^t$.

No living population can follow unrestricted exponential growth forever. Food, water, habitat, oxygen, waste accumulation, competition, predation, disease, climate, and social conditions alter the rate. Logistic and Gompertz models represent slowing growth; Ricker and Beverton-Holt models represent season-to-season reproduction; matrix models track ages or life stages; predator-prey and epidemic equations track interacting groups. In microbiology, doubling can approximate the exponential phase, but real cultures also experience lag, stationary, and decline phases. In ecology, carrying capacity is useful but it may move as the environment changes.^{[3][4][5]}

FOR HUMAN POPULATIONS

Use a constant-rate formula for short illustrations. Use demographic accounting or cohort-component models when age structure, fertility, mortality, and migration matter.

FOR OTHER LIVING SYSTEMS

Yes, the formulas apply - after the time step, reproductive process, resource limit, life stage, and species interactions are defined correctly.

ARTICLE CONCLUSION - GROWTH IS A PROCESS, NOT A PERMANENT NUMBER

A rate measured in one interval may not continue into the next. Good population work therefore reports a central projection, alternative scenarios, and uncertainty. The most useful question is not only What number does the equation produce? but also Which biological and demographic processes would have to remain true for that number to occur?

Definitions: The Language of Population Models

TERM	DEFINITION	TERM	DEFINITION
Population (P or N)	The number of individuals meeting a stated membership rule at a stated time and place.	Initial population (P ₀)	The starting number at $t = 0$. The subscript 0 means initial, not multiplication by zero.
Time (t)	Elapsed intervals such as years, days, generations, or cell cycles. Units must match the rate.	Discrete rate (r)	Proportional change per time step in $P(t) = P_0(1 + r)^t$. A 2.71% rate means $r = 0.0271$.
Continuous rate (k)	Instantaneous proportional rate in $P(t) = P_0 e^{(kt)}$. It is not generally identical to a discrete r.	Growth factor	The multiplier per step: $1 + r$. For $r = 0.10$, the factor is 1.10.
Euler's number (e)	The constant 2.7182818..., the base of natural logarithms and continuous compounding.	Natural logarithm (ln)	The logarithm to base e; it is used to solve for time, rate, doubling time, or half-life.
Births (B)	New individuals added by birth, hatching, germination, division, or another defined reproductive event.	Deaths (D)	Individuals removed by mortality during the interval.
Immigration (I)	Individuals entering the defined population.	Emigration (E)	Individuals leaving the defined population.
Net migration	Immigration minus emigration: $I - E$.	Per-capita rate	Change divided by the population at risk; a rate per individual per unit time.
Carrying capacity (K)	A model parameter representing the population level an environment can sustain under stated conditions.	Density dependence	A process whose strength changes with crowding, such as competition or contagious disease.
Generation time (g)	Average time between reproductive generations; for binary fission, time between divisions.	Doubling time	Time required for a growing population to multiply by 2 under a constant rate.
Cohort	A group sharing a starting time or demographic characteristic, often birth year and sex.	Population momentum	Continued growth or decline caused by age structure even after fertility changes.
Age/stage structure	The distribution among age classes or biological stages such as seed, juvenile, and adult.	Closed population	A population with no immigration or emigration during the modeled interval.
Model parameter	A quantity such as r, k, K, survival, or fertility that controls model behavior.	Projection	A conditional calculation based on assumptions; it is not a guaranteed prediction.

SYMBOL CHECK

P_0 = initial population; $P(t)$ = population after time t ; r = discrete proportional rate; k = continuous rate; K = carrying capacity; B , D , I , E = births, deaths, immigration, emigration. Always write the units beside every rate.

WRITTEN RATE	DECIMAL	PERIOD FACTOR	PLAIN MEANING
0.1%	0.001	1.001	Add one-tenth of one percent per step.
1%	0.01	1.01	Add one percent per step.
10%	0.1	1.1	Add ten percent per step.
100%	1.0	2.0	Double in one step.
1,000%	10.0	11.0	Add ten times the start; end at eleven times it.

UNIT DISCIPLINE

If t is measured in years, r must be per year. If a bacterial rate is per hour, either express t in hours or convert the rate. Never insert a daily rate into a yearly formula without conversion.

Formula Table I: Counting, Compounding, and Limits

MODEL	FORMULA	WHAT IT ANSWERS	MAIN ASSUMPTION / NOTE
Discrete geometric growth	$P(t) = P_0(1 + r)^t$	Population after t equal steps.	Same proportional rate r in every step.
One-step change	$\Delta P = rP_0$ $P_1 = P_0 + \Delta P$	Increase or decrease during one discrete interval.	Use r as a decimal: 2.71% becomes 0.0271.
Continuous exponential	$P(t) = P_0 e^{kt}$	Continuous proportional change.	k has units of inverse time.
Growth differential equation	$dP/dt = kP$	The instantaneous growth law behind continuous exponential change.	Rate of change is proportional to current P.
Demographic balance	$P_2 = P_1 + B - D + I - E$	Population accounting between two dates.	Events must use the same boundary and interval.
Net discrete rate	$r = (B - D + I - E)/P_1$	Observed net proportional change over one step.	Denominator convention must be stated.
Compound annual rate	$r = (P_t/P_0)^{1/t} - 1$	Constant annual rate connecting two observed endpoints.	Summarizes the interval; does not reveal yearly variation.
Doubling time	$t_d = \ln(2)/\ln(1 + r)$ continuous: $t_d = \ln(2)/k$	Time to double under constant growth.	Requires positive rate; units follow the rate.
Logistic differential model	$dP/dt = kP(1 - P/K)$	Growth that slows as P approaches K.	K and k are treated as fixed parameters.
Logistic solution	$P(t) = K / [1 + ((K-P_0)/P_0)e^{-kt}]$	Population at time t under continuous logistic growth.	S-shaped trajectory for $0 < P_0 < K$.

DISCRETE AND CONTINUOUS RATE CONVERSION

Equivalent rates satisfy $1 + r = e^k$. Therefore $k = \ln(1 + r)$ and $r = e^k - 1$.

DECLINE IS INCLUDED

If $-1 < r < 0$, then $0 < 1 + r < 1$ and the geometric model decreases. Example: $r = -0.02$ means a 2% loss per step.

WORKED: SOLVE FOR TIME

If $P_t/P_0 = 3$ and $r = 0.02$, then $t = \ln(3)/\ln(1.02) = 55.48$ periods. Logarithms bring the unknown exponent down where it can be solved.

WORKED: SOLVE FOR RATE

If $P_0 = 10,000$ and $P_{20} = 15,000$, then $r = (1.5)^{1/20} - 1 = 0.02048$, or about 2.048% per period.

ASSUMPTION CHECK BEFORE USING A CLOSED-FORM FORMULA

Confirm: (1) population boundary; (2) starting date; (3) time-step units; (4) whether migration is included; (5) whether the rate is discrete or continuous; (6) whether r and K remain constant; and (7) whether the output represents people, organisms, colonies, cells, biomass, or another unit.

HALF-LIFE FOR A DECLINING POPULATION

When $-1 < r < 0$, solve $(1+r)^t = 1/2$: $t_{\text{half}} = \ln(0.5)/\ln(1+r)$. For $r = -0.02$ per year, the half-life is about 34.31 years.

Formula Table II: Life Cycles, Crowding, and Interaction

MODEL	FORMULA	COMMON USE
Binary fission	$N = N_0 2^n = N_0 2^{t/g}$	Bacteria or cells during an idealized doubling phase.
Gompertz	$N(t) = K \exp[-A e^{-kt}]$	Asymmetric S-shaped growth in microbes, tissues, tumors, or organisms.
Ricker	$N_{t+1} = N_t \exp[r(1 - N_t/K)]$	Seasonal populations; can oscillate when r is large.
Beverton-Holt	$N_{t+1} = R_0 N_t / [1 + (R_0 - 1)N_t/K]$	Density-limited recruitment in fisheries and animal populations.
Allee effect	$dN/dt = rN(1 - N/K)(N/A - 1)$	Low-density difficulty finding mates or cooperating; A is a threshold.
Stage/age matrix	$n(t+1) = L n(t)$	Humans, animals, and plants with different fertility and survival by stage.
Predator-prey	$dN/dt = rN - aNY$ $dY/dt = baNY - mY$	Interacting prey N and predator Y .
SIR epidemic	$dS/dt = -\beta SI/N$ $dI/dt = \beta SI/N - \gamma I$ $dR/dt = \gamma I$	Movement among susceptible, infectious, and removed compartments.
Constant net migration	$dP/dt = kP + M$	Continuous natural growth plus a constant migration flow M .

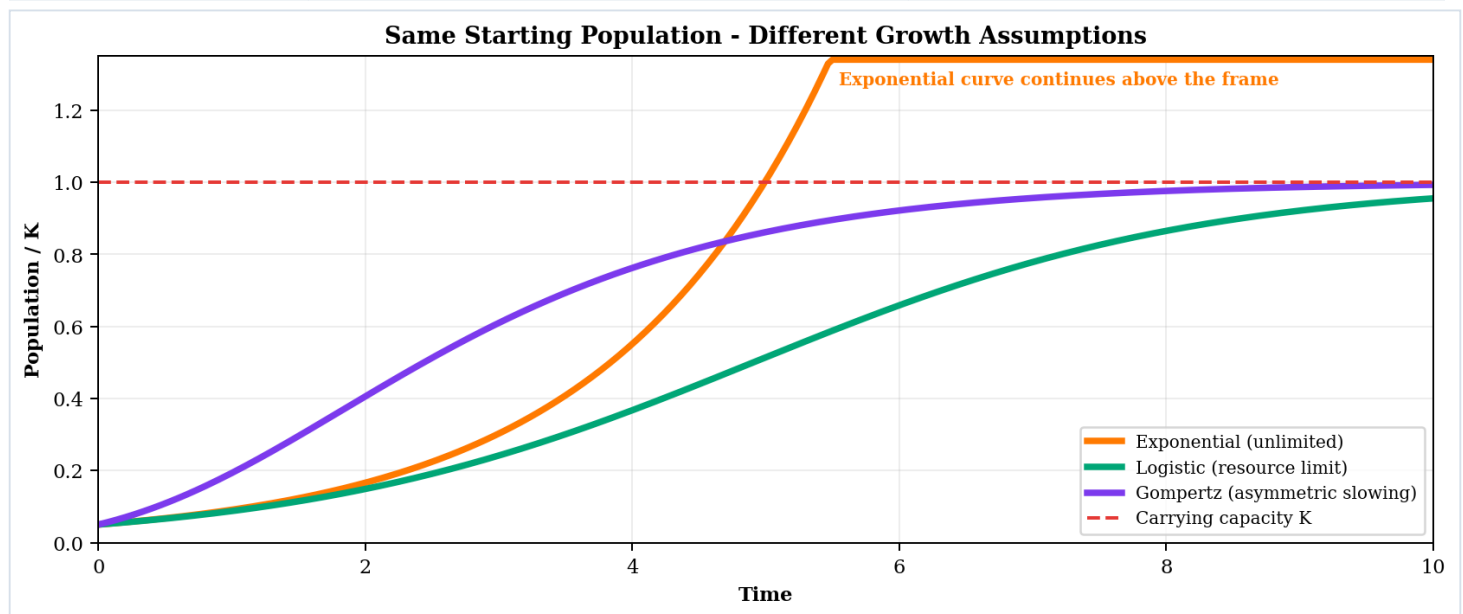


Figure 2. Exponential, logistic, and Gompertz curves can start at the same P_0 yet separate because they represent different assumptions about limitation and slowing.

MODEL CHOICE IS A SCIENTIFIC DECISION

A more complicated equation is not automatically better. Choose the simplest model that respects the biological question, time scale, available data, and mechanism. Then compare projections with observations and revise the parameters.

WHEN RANDOMNESS MATTERS

Small populations may change through chance births, deaths, disasters, or founder effects. A stochastic model treats some events as probabilities rather than fixed averages. Two populations with the same starting values and parameters can then follow different paths.

Is the Growth Rate e? A Careful Correction

2.71%

divide by 100

0.0271

0.00271

multiply by 100%

0.271%

e

mathematical constant

2.71828...

Euler's number appears in continuous growth; it is not the same thing as a percent rate.

THE CENTRAL DISTINCTION

$e = 2.7182818...$ is a mathematical constant. It is the natural base that appears when change is continuously proportional to the current amount. A rate of 2.71% is much smaller and must be written 0.0271 in a formula. The decimal 0.00271 means 0.271%.

Worked one-year example: start with $P_0 = 19,000,000$ people.

INTERPRETATION	CALCULATION	ONE-YEAR INCREASE	ENDING POPULATION
Discrete 2.71%	$19,000,000(1 + 0.0271)$	514,900	19,514,900
Discrete 0.271%	$19,000,000(1 + 0.00271)$	51,490	19,051,490
Continuous $k = 0.0271/\text{year}$	$19,000,000 e^{0.0271}$	521,940	19,521,940
Continuous $k = 0.00271/\text{year}$	$19,000,000 e^{0.00271}$	51,560	19,051,560

IF $r = e$ IN THE DISCRETE MODEL

Then $r = 2.71828$ means 271.828% per year. The one-year factor is $1 + e = 3.71828$, so 19,000,000 becomes about 70,647,355.

IF $k = e$ IN THE CONTINUOUS MODEL

Then $P(1) = 19,000,000 e^e$. The one-year factor is about 15.1543, so the result is about 287,930,983.

ANSWER TO THE ONE-YEAR QUESTION

At a true annual rate of 2.71%, the discrete model adds 514,900 people to a starting population of 19 million in the first year. If the intended decimal is 0.00271, the rate is only 0.271% and the first-year addition is 51,490.

RATE PARAMETER	DISCRETE 100-YEAR RESULT	CONTINUOUS 100-YEAR RESULT	APPROX. DOUBLING TIME
0.0271/year	275,439,691	285,556,235	25.92 y discrete; 25.58 y continuous
0.00271/year	24,905,096	24,914,226	256.12 y discrete; 255.77 y continuous

WHY THE DISCRETE AND CONTINUOUS ANSWERS DIFFER

Using the same numeral for r and k does not create equivalent models. To match them exactly, use $k = \ln(1+r)$. For a discrete 2.71% rate, the equivalent continuous parameter is $\ln(1.0271) =$ about 0.026739 per year.

Can These Formulas Describe Other Forms of Life?

Yes - With Care



Figure 3. Population models are portable across biology, but the meaning of an individual, a birth, a death, and a time step must be defined for each system.

SYSTEM	USEFUL STARTING MODELS	WHAT MUST BE ADDED OR CHECKED
Humans	Demographic balance; cohort-component; Leslie matrix; short-term exponential.	Age and sex structure, fertility, mortality, migration, policies, uncertainty, population momentum.
Wild animals	Logistic, Ricker, Beverton-Holt, stage matrix, predator-prey.	Seasonality, territory, food, harvesting, disease, predation, migration, age at reproduction.
Plants and forests	Stage matrix, logistic, competition models.	Seeds, dormancy, germination, size classes, disturbance, light, water, long lifespans.
Bacteria and yeast	Binary fission, exponential, logistic, Gompertz.	Lag phase, nutrients, oxygen, temperature, waste, stationary phase, cell death.
Cells and tissues	Exponential, logistic, Gompertz, branching models.	Cell-cycle variation, differentiation, immune response, treatment, physical space.
Viruses	SIR/SEIR, within-host target-cell models.	Viruses depend on host cells; count virions, infected cells, or cases explicitly.
Multiple species	Lotka-Volterra, food-web, competition, metapopulation models.	Interaction strength, spatial movement, feedback, stochastic events, changing habitat.

ONE EQUATION, DIFFERENT CLOCKS

A human rate may be per year; bacteria may be per minute; insects may be per season; trees may be tracked by life stage. Changing units without changing the rate changes the answer.

INDIVIDUALS ARE NOT ALWAYS SIMPLE

A clonal colony, fungal network, plant genet, viral particle, or social insect colony may require a carefully chosen unit of counting.

FOUR MODEL-SELECTION QUESTIONS

1) What is counted? Individuals, cells, colonies, biomass, or infected hosts? 2) What changes the count? Reproduction, death, movement, or stage transition? 3) What limits growth? Resources, crowding, predators, disease, or host availability? 4) What data exist? A model cannot estimate more separate parameters than the observations can support.

Starting at 19 Million: Five Constant Annual Rates for 100 Years

EXPERIMENT SETUP - NOT A HISTORICAL CLAIM

Let $P_0 = 19,000,000$ in 1933 and $t = 100$ years in 2033. For each row, use $P(100) = 19,000,000(1 + r)^{100}$. The rates are intentionally held constant to reveal compounding. Real human populations do not keep extreme rates unchanged for a century.

r	ANNUAL %	1+r	100-YEAR FACTOR	POPULATION IN 2033	DOUBLING TIME*
0.001	0.1%	1.001	1.10512	20,997,198	693 years
0.01	1%	1.01	2.70481	51,391,463	69.7 years
0.1	10%	1.1	13780.6	261,831,634,457	7.27 years
1	100%	2	1.268×10^{30}	2.409×10^{37}	1 years
10	1000%	11	1.378×10^{104}	2.618×10^{111}	0.289 years

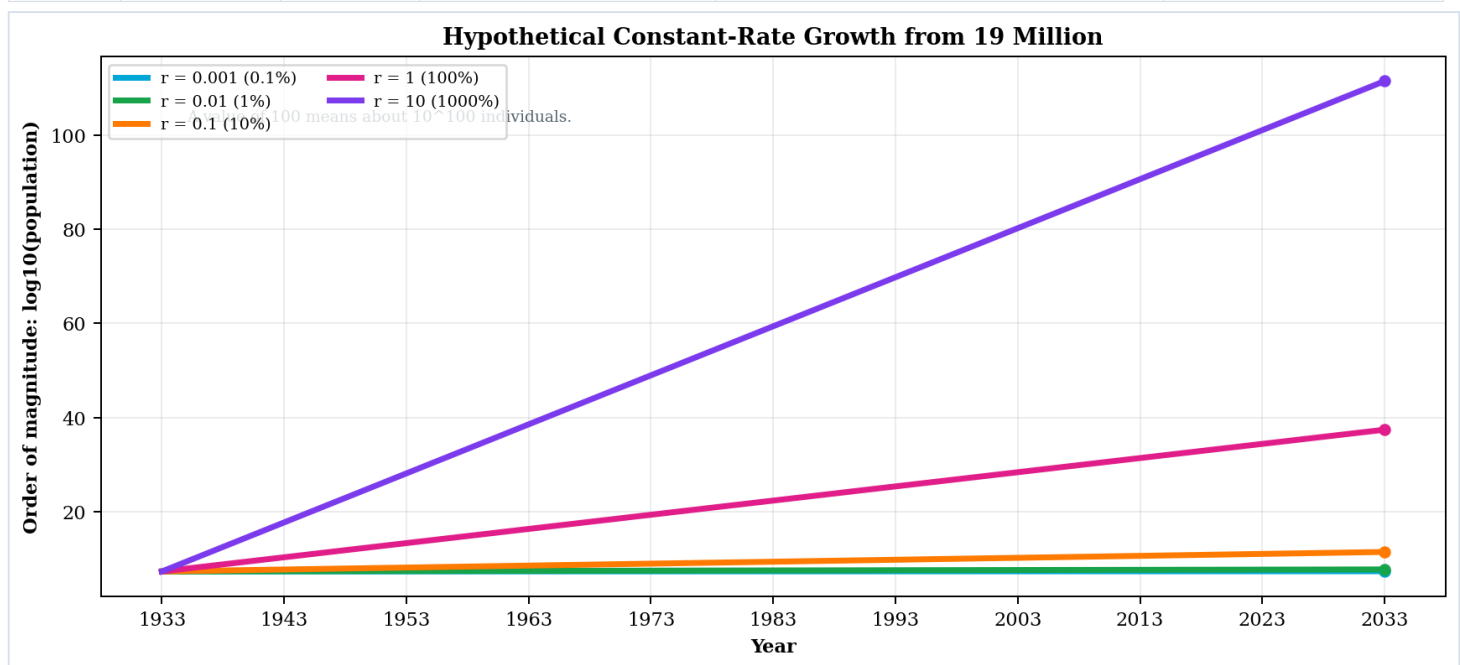


Figure 4. The vertical scale shows $\log_{10}(\text{population})$, so equal vertical distances represent factors of 10. This is the only practical way to place all five scenarios on one graph.

READ THE EXTREMES CORRECTLY

At $r = 1.0$, the population doubles every year: 2^{100} times the start. At $r = 10.0$, the annual factor is 11, not 10: the population is multiplied by 11 one hundred times. These are mathematical stress tests, not plausible century-long human rates. *Doubling time for annual discrete models is an interpolated equivalent.

ASSUMPTIONS HELD FIXED IN EVERY ROW

No migration shock, war, epidemic, fertility transition, age-structure change, resource ceiling, environmental feedback, or policy change alters r . That deliberate simplification makes the mathematics transparent while making the extreme rows biologically unrealistic.

The Same Five Scenarios, Decade by Decade

YEAR	t	r=.001	r=.01	r=.1	r=1.0	r=10.0
1933	0	19,000,000	19,000,000	19,000,000	19,000,000	19,000,000
1943	10	19,190,857	20,987,820	49,281,107	19,456,000,000	4.928×10^{17}
1953	20	19,383,632	23,183,611	127,822,499	1.992×10^{13}	1.278×10^{28}
1963	30	19,578,343	25,609,129	331,538,643	2.040×10^{16}	3.315×10^{38}
1973	40	19,775,009	28,288,411	859,925,856	2.089×10^{19}	8.599×10^{48}
1983	50	19,973,652	31,248,005	2,230,426,205	2.139×10^{22}	2.230×10^{59}
1993	60	20,174,290	34,517,237	5,785,151,151	2.191×10^{25}	5.785×10^{69}
2003	70	20,376,943	38,128,504	15,005,192,179	2.243×10^{28}	1.501×10^{80}
2013	80	20,581,632	42,117,589	38,919,604,077	2.297×10^{31}	3.892×10^{90}
2023	90	20,788,377	46,524,021	100,947,429,625	2.352×10^{34}	1.009×10^{101}
2033	100	20,997,198	51,391,463	261,831,634,457	2.409×10^{37}	2.618×10^{111}

SLOW LOOKS ALMOST LINEAR AT FIRST

At $r = 0.001$, a 0.1% annual rate turns 19.0 million into about 21.0 million after a century - a total factor of only 1.105.

MODERATE RATES BECOME LARGE

At $r = 0.10$, each decade multiplies the population by $1.1^{10} = 2.5937$. After 100 years the factor is 13,780.6.

EXPONENTS MULTIPLY ORDERS OF MAGNITUDE

Take logarithms: $\log_{10} P(t) = \log_{10} P_0 + t \log_{10}(1+r)$. The right side is a straight line in t when r is constant. For $r = 10$, each year adds $\log_{10}(11)$ about 1.0414 to the exponent; after 100 years the exponent increases by about 104.14.

Interpretation: The formula is doing exactly what it was asked to do, but a result can be mathematically correct and biologically impossible. An enormous output is a warning to inspect the assumptions - especially whether a constant rate, unlimited resources, fixed boundaries, and unchanged time units make sense.

ANNUAL RATE	10-YEAR FACTOR $(1+r)^{10}$	100-YEAR FACTOR	WHAT REPEATS EACH DECADE
0.1%	1.010045	1.105116	Each decade adds about 1.0045% to the current amount.
1%	1.104622	2.704814	Each decade multiplies by about 1.1046.
10%	2.593742	13,780.612340	Each decade multiplies by about 2.5937.
100%	1,024	1.2677×10^{30}	Ten doublings per decade.
1,000%	2.5937×10^{10}	1.3781×10^{104}	Ten multiplications by 11 per decade.

SCIENTIFIC NOTATION

The form $a \times 10^n$ places one nonzero digit before the decimal. The exponent n tells the order of magnitude. Example: 2.41×10^{37} has 38 digits before the decimal point.

PLAUSIBILITY CHECK

Compare the projected count with land, energy, food, reproduction time, habitat, and the age distribution. A contradiction means the assumptions break before the arithmetic does.

What Changes When the Environment Has a Limit?

Suppose the 1933 population still starts at 19 million, but the environment is represented by a carrying-capacity parameter $K = 60$ million and continuous intrinsic rate $k = 0.03/\text{year}$. Compare the unrestricted exponential model with the continuous logistic model. This is an illustration, not an estimate of any named population.

UNRESTRICTED EXPONENTIAL

$$P(t) = 19e^{0.03t} \text{ million}$$

The growth rate remains proportional to P and never slows because of crowding.

LOGISTIC WITH $K = 60$ MILLION

$$P(t) = 60 / [1 + (41/19)e^{-0.03t}] \text{ million}$$

The factor $(1 - P/K)$ slows growth near K .

YEAR	t	EXPONENTIAL (MILLIONS)	LOGISTIC (MILLIONS)	LOGISTIC GROWTH PRESSURE $1-P/K$
1933	0	19.000	19.000	0.683
1958	25	40.223	29.713	0.505
1983	50	85.152	40.500	0.325
2008	75	180.267	48.882	0.185
2033	100	381.625	54.179	0.097

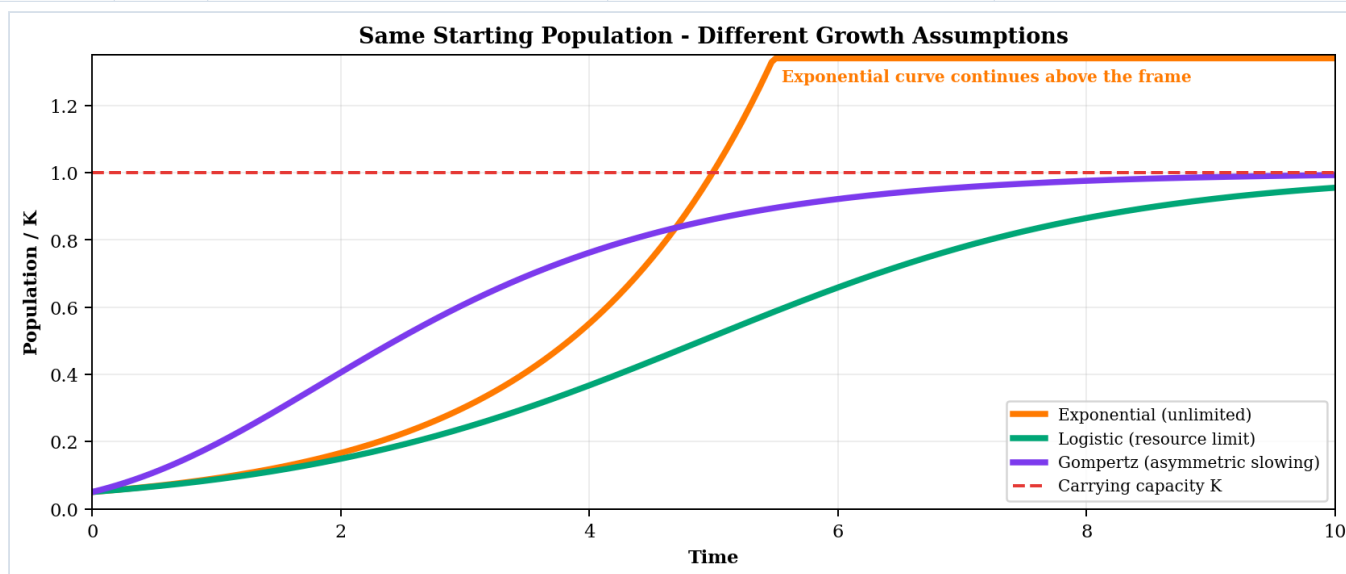


Figure 5. Resource-limited curves bend as growth slows. The logistic curve is symmetric around its fastest-growth point; the Gompertz curve slows asymmetrically.

A CARRYING CAPACITY IS NOT A PERMANENT CEILING

K summarizes conditions inside a model. Technology, habitat restoration, drought, pollution, conflict, disease, climate, food systems, and migration can change the effective limit. A model with a time-varying K may be more appropriate than a fixed K .

Microorganisms, Animals, and Plants

BINARY FISSION: $N = N_0 \times 2^{(t/g)}$

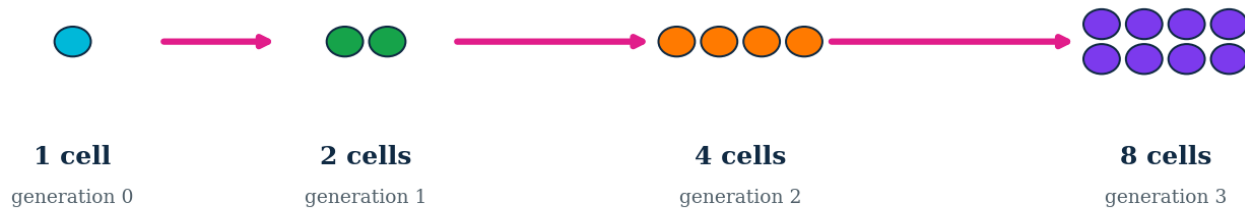


Figure 6. One ideal binary-fission generation doubles the count. Repeating the process n times gives the factor 2^n .

EXAMPLE A - BACTERIAL DOUBLING

A culture begins with $N_0 = 1,000$ cells and doubles every $g = 30$ minutes. In 24 hours, $n = 1,440/30 = 48$ generations. The unrestricted model gives $N = 1,000 \times 2^{48} = 2.8147 \times 10^{17}$ cells. This enormous result explains why nutrient limits, waste, space, oxygen, and cell death must eventually replace the unlimited model.

EXAMPLE B - ANIMAL POPULATION

Let $N = 400$ deer, $K = 1,000$, and $k = 0.5/\text{year}$. Logistic instantaneous growth is $dN/dt = 0.5(400)(1 - 400/1000) = 120$ deer/year at that moment. It is a rate, not necessarily a whole-number census result.

EXAMPLE C - TWO-STAGE PLANTS

Let $n = [100 \text{ seeds}, 40 \text{ adults}]$. If $L = [[0.4], [0.3, 0.6]]$, then next year's vector is $Ln = [160 \text{ seeds}, 54 \text{ adults}]$. The matrix separates reproduction, germination, and adult survival.

EXAMPLE D - INTERACTING SPECIES

A prey-only exponential model may fail when predators are important. The term $-aNY$ removes prey at a rate proportional to encounters between prey N and predators Y . The paired predator equation adds predators through consumed prey and removes them through mortality. Interaction terms turn a single curve into a coupled system.

Measurement lesson: Microbial counts may mean cells, colony-forming units, optical density, gene copies, or biomass. Animal counts may come from captures, tracks, cameras, or statistical estimates. A formula cannot repair an unclear measurement definition.

MICROBIAL PHASE	TYPICAL PATTERN	WHY THE SIMPLE DOUBLING MODEL CHANGES
Lag	Cells adjust; little net increase.	Enzymes and cell machinery are being prepared.
Exponential	Approximately constant doubling time.	$N = N_0 2^{(t/g)}$ may be useful over a limited interval.
Stationary	Births roughly balance deaths.	Nutrients or space are limited; wastes accumulate.
Decline	Viable count decreases.	Mortality exceeds reproduction.

VALIDATE A BIOLOGICAL MODEL

Plot residuals, compare predicted and observed counts, reserve data for testing, report parameter uncertainty, and ask whether the fitted parameter values are biologically possible. A visually smooth curve is not proof that the mechanism is correct.

Student Exercises: Rates, Models, and Meaning

Show units, write percentages as decimals before substituting, and identify every assumption.

1. Convert 1.8% to a decimal rate. Then convert 0.0045 to a percent.

2. Explain why 2.71% is 0.0271 rather than 0.00271.

3. A population starts at 50,000 and grows 2% annually for 5 years. Find $P(5)$.

4. Find the discrete doubling time for $r = 0.01$ per year.

5. For $P_0 = 19,000,000$ and $r = 0.001$, find the first-year increase and P_1 .

6. A city begins with 100,000 people. During one year it has $B=1,800$, $D=1,000$, $I=700$, $E=400$. Find the new population and net rate.

7. A culture starts with 500 cells and completes 8 ideal doublings. Find N .

8. In a logistic model, $N=400$, $K=1,000$, $k=0.5/\text{year}$. Compute dN/dt .

9. Choose exponential or logistic: a yeast culture grows in a sealed container until nutrients become scarce. Explain.

10. Convert a 3% discrete annual rate to the equivalent continuous rate $k = \ln(1+r)$.

11. Write a discrete model for a population of 80,000 declining 2% per year. Find the 10-year value.

12. Explain why $r=10.0$ may be mathematically valid in the formula but biologically unreasonable for humans over 100 years.

EXTENSION CHALLENGE

Using the 1933-2033 setup, solve for the constant annual rate that would turn 19 million into 60 million in 100 years: $r = (60/19)^{1/100} - 1$. Then compare it with the continuous rate $k = \ln(60/19)/100$.

FORMULA BANK	USE
$P(t) = P_0(1+r)^t$	Discrete constant-rate growth or decline.
$P(t) = P_0 e^{(kt)}$	Continuous constant-rate growth or decline.
$P_2 = P_1 + B - D + I - E$	Direct demographic accounting.
$dP/dt = kP(1-P/K)$	Continuous resource-limited growth.
$N = N_0 2^{(t/g)}$	Ideal binary doubling.

QUICK SELF-CHECK

A complete solution states the model, substitutes the decimal rate, shows the time units, rounds only at the end, interprets the number in words, and names at least one assumption that could fail.

Answer Key, Key Notes, and Scientific Sources

#	ANSWER
1	$1.8\% = 0.018$; $0.0045 = 0.45\%$.
2	Percent means per 100: $2.71/100 = 0.0271$. The decimal 0.00271 multiplied by 100% is 0.271%.
3	$P(5)=50,000(1.02)^5 = 55,204.04$, about 55,204.
4	$td=\ln 2/\ln 1.01 = 69.6607$ years.
5	Increase = $19,000,000(0.001)=19,000$; $P1=19,019,000$.
6	$P2=100,000+1,800-1,000+700-400=101,100$. Net rate= $1,100/100,000=0.011=1.1\%$.
7	$N=500 \times 2^8 = 128,000$ cells.
8	$dN/dt=0.5(400)(1-0.4)=120$ deer/year at that instant.
9	Exponential may approximate the early phase; logistic or Gompertz is more suitable once resources become limiting.
10	$k=\ln(1.03)=0.0295588/\text{year}$, about 2.956% as a continuous rate parameter.
11	$P(t)=80,000(0.98)^t$; $P(10)=65,365.82$, about 65,366.
12	$r=10$ means a 1,000% annual rate and factor 11 every year. Resource limits, age structure, reproduction time, mortality, and changing rates make century-long human use implausible.
Ext.	Discrete $r=(60/19)^{(1/100)}-1=0.0115654$, or 1.15654%/year. Continuous $k=\ln(60/19)/100=0.0114991/\text{year}$, or about 1.14991% as a continuous parameter.

TEN KEY NOTES

1) Define the population boundary. 2) Match rate units to time units. 3) Convert percent to decimal. 4) Distinguish discrete r from continuous k . 5) e is a constant, not a percent. 6) Exponential growth assumes a constant proportional rate. 7) Logistic growth adds a limiting factor. 8) Human projections require births, deaths, migration, and age structure. 9) Biological models must be checked against data. 10) A projection is conditional on its assumptions.

SCIENTIFIC AND EDUCATIONAL SOURCES

[1] United Nations, [World Population Prospects 2024: Methodology of the United Nations Population Estimates and Projections](#).

[2] U.S. Census Bureau, [The Components of Demographic Change](#).

[3] OpenStax Biology 2e, [Environmental Limits to Population Growth](#).

[4] OpenStax Biology 2e, [Human Population Growth](#).

[5] Allen and Waclaw, [Bacterial Growth: A Statistical Physicist's Guide](#), Reports on Progress in Physics (2019), PMC6330087.

[6] OpenStax Calculus Volume 2, [The Logistic Equation](#).

[7] U.S. Census Bureau, [Population Estimates and Projections: Methods, Providers, and Characteristics](#).

[8] NCBI Bookshelf, [Medical Microbiology: bacterial structure and binary fission](#).

Sources accessed September 24, 2026. All numerical case-study outputs in this guide were calculated directly from the stated formulas and assumptions.

IF THE QUESTION EMPHASIZES...	BEGIN WITH...
A constant percent per year	Discrete geometric or continuous exponential growth.
Births, deaths, and movement	Demographic balancing equation.
Age, sex, or life stage	Cohort-component or matrix model.
Crowding or limited resources	Logistic, Gompertz, Ricker, or Beverton-Holt.
Two interacting populations	Predator-prey, competition, or epidemic compartments.

FINAL THOUGHT

Population equations are lenses. They help us see how repeated proportions, demographic flows, resources, life cycles, and interactions shape change. Their power comes from using them transparently - with correct units, stated assumptions, measured data, and respect for the living systems behind the numbers.